[illegible]

LOVELL'S SERIES OF SCHOOL BOOKS

ELEMENTARY ARITHMETIC,

IN DECIMAL CURRENCY;

[illegible]

CANADIAN SCHOOLS.

JOHN HERBERT SANGSTER, M.A., M.D.

[illegible]

[illegible]

MONTREAL:

PRINTED AND PUBLISHED BY JOHN LOVELL.

TORONTO:

R. & A. MILLER, 62 KING ST[illegible]

[illegible]

[illegible]

460

H04

Cream Tartar Halfe Pound
4 Lemmons
1 Pint Vinegar
1 Galle Rain Water
Sponge the face & Neck

this book belongs to
miss margreatgowins
Grest Grave
library

Miss Margreat
Owens

27

e Q J maria

hungartowen

look beyond the

light for the

J E B C C

LOVELL'S SERIES OF SCHOOL BOOKS.

ELEMENTARY ARITHMETIC,

IN DECIMAL CURRENCY;

DESIGNED FOR THE USE OF

CANADIAN SCHOOLS.

BY

JOHN HERBERT SANGSTER, M.A., M.D.

MATHEMATICAL MASTER AND LECTURER IN CHEMISTRY AND NATURAL PHILOSOPHY IN THE NORMAL SCHOOL FOR UPPER CANADA.

THIRD EDITION.

Montreal:
PRINTED AND PUBLISHED BY JOHN LOVELL,
AND FOR SALE AT THE BOOKSTORES.
1866.

Entered, according to the Act of the Provincial Parliament, in the year one thousand eight hundred and sixty, by John Lovell, in the Office of the Registrar of the Province of Canada.

JOHN F. TROW,
ELECTROTYPER,
NEW YORK.

PREFACE.

In presenting the Elementary Arithmetic to his fellow Canadian Teachers, the author respectfully solicits their attention to the following brief explanation of its arrangement and design.

First, then, with regard to the purpose it is designed to serve, it may be remarked that the Elementary Arithmetic is eminently a *practical* treatise on numbers. Every rule in the book is expressed as clearly and as concisely as possible; is then illustrated by two or more examples worked out and fully explained; and is finally enforced by an exercise containing a sufficient number of problems to thoroughly impress it upon the pupil's memory. This latter object is still more completely attained by the miscellaneous or review problems scattered through the work. It will, however, be observed, that, with the exception of Notation, Numeration, the Simple Rules, and Decimal Money, no attempt has been made to give the pupil worded reasons for the processes employed; that, except so far as the several rules are explained by the examples solved, the work of illustrating and explaining is left to the teacher. This plan has been adopted chiefly from two considerations. In the first place, young children, those for whom the work is primarily intended, learn the reasons of the rules far more easily and expeditiously from familiar and repeated illustrations by the teacher on the blackboard, than they can by studying printed demonstrations; and, in the second place, had these reasons and explanations been inserted, they would have increased the size of the book far beyond what was considered desirable.

It is however believed that in the greater number of instances the rule is so worded, and the solution and accompanying explanation of the two or three illustrative examples are so given, as to enable the pupil to master and comprehend the *rationale* of the process employed. This remark does not, of course, apply to the extraction of the square and cube roots, but it holds with regard to almost every other rule in the book. For a full elucidation and discussion of the principles involved in arithmetical operations, the attention of the more advanced student is respectfully directed to the author's National Arithmetic.

With respect to the arrangement, a few words will suffice. In commencing the Elementary the pupil is assumed to have no previous knowledge of arithmetic, and accordingly great care has been expended in wording the definitions, explanations, rules, &c., as concisely as possible, and in making preliminary problems of the very easiest description. The author has also endeavored, at those parts of the subject at which the pupil invariably meets with more or less trouble and difficulty, to prepare him for the consideration of the rule and the solution of problems on the slate by a series of simple mental exercises. It is not for a moment presumed that these mental exercises contain all that is necessary in the way of preparation: they are rather designed to serve as a *sample* of the introductory drilling through which the class should enter the rule. The judicious teacher will continue some such exercise as a mental training until he is convinced that his pupils can enter into the solution of questions on the slate without any such miserable artifices as the attempt to aid their ability to add or subtract by counting on their fingers or on the notches cut in their slate frames.

The teacher is earnestly recommended to begin, at as early a period as practicable, drilling the pupils on the Mental Arithmetic at the end of the book. He will find it the most efficient of all means for calling forth and cultivating the intellectual faculties of his scholars, and at the same time the most unfailing and successful mode of making them thoroughly comprehend the principles of written arithmetic. Although the mental exercises alluded to contain a large number of problems, it is taken for granted the teacher will not confine his class to these, but will from time to time supply them with similar questions of his own construction.

The problems throughout the book are all new, and no pains have been spared in reading the proof-sheets to ensure the most rigid accuracy in every part.

TORONTO, May, 1860.

CONTENTS.

SECTION I.

PAGE
Definitions 9
Numeration 10
Numeration Table 12
Notation 13
Roman Notation 15
Recapitulation and Examination Questions on Numeration and Notation 17
Simple Addition 19
Recapitulation and Examination Questions on Addition 26
Simple Subtraction 27
Recapitulation and Examination Questions on Subtraction 32
Simple Multiplication 33
Multiplication Table 34
To Multiply by a number not greater than 12 35
To Multiply by a Composite Number 36
To Multiply by a number greater than 12 and not composite 37
Proof of Multiplication 38
Recapitulation and Examination Questions on Multiplication 40
Simple Division 42
Short Division 44
To Divide by a Composite Number 45
Long Division 46
Proof of Division 46
Recapitulation and Examination Questions on Division 48

SECTION II.

PAGE
Decimal Currency........ 50
To Reduce Old Canadian Currency to Dollars and Cents........ 51
To Reduce Dollars and Cents to Old Canadian Currency........ 52
Addition, Subtraction, Multiplication, and Division of Decimal Money........ 53
Examination Questions on Decimal Money........ 55
Tables of Money, Weights, and Measures........ 56
Reduction Descending........ 62
Reduction Ascending........ 63
Compound Addition........ 65
Compound Subtraction........ 68
Compound Multiplication........ 70
Compound Division........ 75
To Divide by an Applicate Number........ 77
Miscellaneous Problems on Sections I., II........ 78

SECTION III.

Greatest Common Measure........ 81
Least Common Multiple........ 82

SECTION IV.

Vulgar Fractions—Definitions, &c........ 85
Reduction of Vulgar Fractions........ 88
To Reduce a Mixed Number to an Improper Fraction.. 89
To Reduce an Improper Fraction to a Mixed Number 89
To Reduce a Fraction to its Lowest Terms........ 90
To Reduce Fractions to a Common Denominator........ 91
To Reduce a Compound Fraction to a Simple Fraction 93
To Reduce a Complex Fraction to a Simple Fraction... 94

PAGE
To Reduce a Denominate Fraction from one denomination to another........ 96
To Reduce one Denominate Number to the Fraction of another Denominate Number........ 97
To Find the Value of a Denominate Fraction........ 99
Addition of Fractions........ 100
Subtraction of Fractions........ 101
Multiplication of Fractions........ 103
Division of Fractions........ 105
To Multiply or Divide an Integral Denominate Number by a Fraction........ 106–7
Decimals—Definitions, &c........ 108
Numeration of Decimals........ 108
Notation of Decimals........ 109
Addition and Subtraction of Decimals........ 111
Multiplication of Decimals........ 112
Division of Decimals........ 113
To Reduce a Vulgar Fraction to a Decimal........ 115
Circulating Decimals,—Definitions, &c........ 116
To Reduce a Pure Repetend to a Vulgar Fraction........ 117
To Reduce a Mixed Repetend to a Vulgar Fraction........ 117
Addition, Subtraction, Multiplication, and Division of Circulating Decimals........ 118
To Reduce a given Denominate Number to the Decimal of another Denominate Number........ 119
To Find the Value of a Decimal of a Denominate Number........ 120
Miscellaneous Problems on Sections I.–IV........ 121

SECTION V.

Ratio........ 123
Simple Proportion........ 126
Compound Proportion........ 132

SECTION VI.

Practice........ 136

SECTION VII.

PAGE
Percentage 140
Commission and Brokerage........ 143
Insurance........ 144
Buying and Selling Stocks........ 146

SECTION VIII.

Simple Interest........ 149
To Find Interest at 6 per cent........ 151
Compound Interest........ 153
Discount 155
Bank Discount........ 156
Simple Partnership........ 157
Compound Partnership........ 159

SECTION IX.

Profit and Loss........ 161
Barter 166
Exchange of Currencies........ 168
Analysis........ 171

SECTION X.

Involution 175
Extraction of Square Root........ 177
Extraction of Cube Root........ 180
Miscellaneous Problems........ 182
Mental Arithmetic........ 186
Answers to Exercises........ 201

ARITHMETIC.

SECTION I.

DEFINITIONS, NUMERATION, SIMPLE ADDITION, SIMPLE SUBTRACTION, SIMPLE MULTIPLICATION, AND SIMPLE DIVISION.

1. Arithmetic is the study of numbers.

2. Numbers are expressions or characters that represent one or more things of the same kind. Thus *one*, *two*, *three*, *seven*, *eleven*, *twenty-six*, &c., are numbers.

3. Numbers may be expressed either by words or by characters.

4. Notation is the art of writing numbers by means of characters.

5. Numeration is the art of reading numbers thus expressed.

6. The characters used for the expression of numbers are either figures or letters.

7. Arabic Notation is the expression of numbers by figures.

8. Roman Notation is the expression of numbers by letters.

9. The figures employed in writing numbers are as follows:—

1, called *one.*
2, " *two.*
3, " *three.*
4, " *four.*
5, " *five.*
6, " *six.*
7, " *seven.*
8, " *eight.*
9, " *nine.*
0, " *naught, nothing, cipher, or zero.*

10. All numbers higher than *nine* are represented by writing two or more of these figures together.

Thus,	Ten	is written	10
	Eleven	"	11
	Twelve	"	12
	Thirteen	"	13
	Twenty	"	20
	Twenty-one	"	21
	Twenty-two	"	22
	Thirty	"	30
	Thirty-one	"	31
	Forty	"	40
	Fifty	"	50
	One Hundred	"	100
	One Hundred and Ten	"	110
	One Hundred and Eleven	"	111

11. When a number consists of several figures:

The first or right-hand figure is called the units' figure, and is said to stand in the *units' place.*

The second figure from the right hand is called the tens' figure, and is said to stand in the *tens' place.*

The third figure from the right hand is called the hundreds' figure, and is said to stand in the *hundreds' place.*

The fourth figure is called the thousands' figure, and is said to stand in the *thousands' place*, &c.

12. The figures 1, 2, 3, 4, 5, 6, 7, 8, and 9, are called *Significant Figures*, because each of them represents or stands for one or more individual things. They are also called *digits*, from a Latin word meaning "finger," because many uneducated persons are in the habit of counting on the fingers.

13. The character 0 is itself of no value, and is used merely to change the value of the significant figures by making them occupy different places. Thus 7 standing alone is *seven*, followed by one cipher it is 70 or *seventy*, by two ciphers it is 700 or *seven hundred*, by three ciphers it is 7000 or *seven thousand*, &c.

EXERCISE 1.

1. Write neatly on your slate all the numbers from 1 to 100.
2. Write neatly on your slate all the numbers from 100 to 200.
3. Read the following numbers: 27, 164, 19, 91, 107, 789, 426, 999.
4. Read the following numbers: 16, 13, 12, 61, 31, 21, 409, 717, 800.
5. Write the following numbers: twenty-eight; five hundred and seventeen; eleven; sixty-five; two hundred and nine; forty; nineteen.
6. Write the following numbers: one hundred and thirty-seven; nine hundred and six; seventy-one; eight hundred and seven; two hundred and fifty.
7. Read the following numbers: 163, 403, 701, 808, 917, 800, 711.
8. Write the following numbers: seventy-nine; eight hundred and forty; seven hundred and eleven; four hundred and sixteen; five hundred and five.
9. Read the following numbers: 909, 81, 17, 111, 606, 510, 170, 919.
10. Write the following numbers: fifty-nine; seventeen; seventy-one; nineteen; nine hundred and forty; sixty-one; four hundred and twelve.

14. To facilitate the reading of large numbers, they are divided into *periods* of three figures each, beginning at the right-hand side.

15. The names of the periods are as follows:—

The first or right-hand period is that of Units.
The second period is that of Thousands.
The third " " Millions.
The fourth " " Billions.
The fifth " " Trillions.

And so on according to the following—

NUMERATION TABLE.

Period	Place	Figure	Period number
SEPTILLIONS.	Hundreds of Septillions.	9	9TH PERIOD
	Tens of Septillions.	8	
	Septillions.	7	
SEXTILLIONS.	Hundreds of Sextillions.	6	8TH PERIOD
	Tens of Sextillions.	5	
	Sextillions.	4	
QUINTILLIONS.	Hundreds of Quintillions	3	7TH PERIOD
	Tens of Quintillions.	2	
	Quintillions.	1	
QUADRILLIONS.	Hundreds of Quadrillions.	9	6TH PERIOD
	Tens of Quadrillions.	8	
	Quadrillions.	7	
TRILLIONS.	Hundreds of Trillions.	6	5TH PERIOD
	Tens of Trillions.	5	
	Trillions.	4	
BILLIONS.	Hundreds of Billions.	3	4TH PERIOD
	Tens of Billions.	2	
	Billions.	1	
MILLIONS.	Hundreds of Millions.	9	3D PERIOD
	Tens of Millions.	8	
	Millions.	7	
THOUSANDS.	Hundreds of Thousands.	6	2D PERIOD
	Tens of Thousands.	5	
	Thousands.	4	
UNITS.	Hundreds.	3	1ST PERIOD
	Tens.	2	
	Units.	1	

16. To read any number it is necessary to remember—

1st. The names of the periods in their order.

2d. That each period consists of so many *hundreds*, *tens*, and *units*.

EXAMPLE 1.—Read the number 742679.

Here we place a comma between the 2 and the 6 and thus divide the number into two periods, thus 742,679, of which the left-hand one is that of thousands and the other that of units. Then, reading each period separately, we find that there are seven hundred and forty-two thousand and six hundred and seventy-nine units, and, reading the whole together—seven hundred and forty-two thousand, six hundred and seventy-nine.

EXAMPLE 2.—Read the number 670493278900.

Here dividing into periods, we get 670,493,278,900, i. e. four periods, the names of which, beginning at the lowest or right-hand period, are units, thousands, millions, and billions.

The 4th period is six hundred and seventy billions.
The 3d period is four hundred and ninety-three millions.
The 2d period is two hundred and seventy-eight thousand; and
The 1st period is nine hundred units.

Then, reading these together, we find that the number is —six hundred and seventy billions, four hundred and ninety-three millions, two hundred and seventy-eight thousand, nine hundred.

Example 3.—Read the number 67040000000007.

Here pointing off into periods we get

67,040,000,000,007,

i. e. five periods, the names of which, beginning at the lowest, are units, thousands, millions, billions, and trillions. Then reading each period separately, we have sixty-seven trillions in the highest period, forty billions in the next, nothing in the next, nothing in the next, and seven in the last. Finally, reading these together, we find that the given number is

Sixty-seven trillions, forty billions, and seven.

Exercise 2.

Read the following numbers :

1. 7040 ; 8101 ; 8000 ; 4070 ; 8019 ; 6111 ; 96003 ; 8674567.
2. 91131140 ; 967004296 ; 61300400007623.
3. 1001001001001 ; 670000000069 ; 8100810081008l.
4. 91234013402 ; 91234267109 ; 100000200003004.
5. 67189456713427 ; 9100009134000671001.
6. 71345671913461300041234.
7. 100001000001000000 ; 203040506070809.
8. 908007000600005 ; 4003000200001.
9. 2046008010 ; 11111111111111.
10. 40007 ; 9000000009 ; 870008700087.

NOTATION.

17. To write down numbers, we must attend to the following—

RULE.

Begin at the left-hand side and write down each period in its proper order, as though it were a period of units.

Place a cipher in each vacant place that occurs in any period; and if any period be wholly vacant, fill it with ciphers.

Example 1.—Write down as one number sixty-seven millions four thousand and eighty-nine.

Here the left-hand period is 67 millions, the next period to the right is 4 thousand, and the last or right-hand period is 89 units. Then writing these together and filling the vacant places in the thousands' and units' periods with ciphers, we get for the required number 67,004,089.

Example 2.—Write down as one number seventeen billions four hundred and twenty-six thousand and one.

Here we begin by writing down 17 billions; this we follow by 000 in the period of millions, this by 426 in the period of thousands, and this by 001 in the period of units. Placing these together we get for the required number 17,000,426,001.

EXERCISE 3.

Write down the following numbers:

1. Three thousand and twenty-nine; five thousand and seventeen; six thousand five hundred; eight thousand and eight; nine thousand two hundred and seven; four thousand and ten; seven thousand and sixty-one; eight thousand seven hundred.
2. Eighty-seven thousand four hundred and eleven; ninety-four thousand and six; thirty thousand four hundred and fifteen; twenty-four thousand and twenty-four; seventy thousand six hundred; thirty thousand and one.
3. Five hundred and sixty-seven thousand; two hundred and four thousand and sixty-three.
4. Seven hundred and sixty-two thousand seven hundred and nine.
5. Six hundred and four thousand and ninety.
6. Seventeen millions and eighty-one; forty millions two thousand and six.
7. One hundred and forty millions six hundred and two thousand and seven; twenty millions and eleven.

8. Eight hundred and seven millions twenty thousand one hundred and ten ; seven hundred millions and twenty thousand.
9. Five billions five millions five thousand and five; twenty billions and one.
10. Sixty trillions sixty millions and sixty.
11. Seventeen trillions seven millions and seventy.
12. Eight billions seventy millions four hundred thousand two hundred and seventy-six.

ROMAN NOTATION.

18. The seven letters used in Roman Notation, with their values, are as follows:

I	*One.*
V	*Five.*
X	*Ten.*
L	*Fifty.*
C	*One Hundred.*
D	*Five Hundred.*
M	*One Thousand.*

19. All other numbers are expressed by repeating or combining these letters, as in the following

TABLE.

I	1	LX	60
II	2	LXX	70
III	3	LXXX	80
IV	4	XC	90
V	5	C	100
VI	6	CC	200
VII	7	CCC	300
VIII	8	CD	400
IX	9	D	500
X	10	DC	600
XI	11	DCC	700
XII	12	DCCC	800
XIII	13	CM	900
XIV	14	M	1000
XX	20	MM	2000
XXX	30	MMM	3000
XL	40	MMMD	3500
L	50	MDCCCLX	1860

20. From this table we learn that—

1st. As often as a letter is repeated its value is repeated, but no letter can be repeated more than thrice.

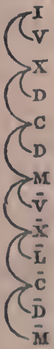
2d. When a letter of a lower value is written before one of a higher, its value is to be subtracted; but the only letters that may be thus written before others are I, X, and C.

3d. When a letter of a lower value is written after one of a higher, their values are to be added.

4th. A bar or a dash written over a letter or combination multiplies its value by 1000. Thus $X = 10$, $\overline{X} = 10000$, $C = 100$, $\overline{C} = 100000$, $CCXV = 215$, $\overline{CCXV} = 215000$, &c.

5th. The characters for 5, 50, and 500 never stand before others of a higher value, and never suffer repetition.

6th. A character can never stand before any other than one of the two next higher in value. Thus I can stand before V or X, but before no other letters; X can stand before L or C. C can stand before D or M; and so on, according to the following scheme:



Exercise 4.

Express the following numbers in Arabic Notation, also read them in common language:

1. IX, XVII, XXXIII, XLIV, XCIX, CDLXXVIII, CCCXXX, XLVII, LXXIV.
2. DCCLXXVII, CCXCVI, DCCCXLIII, CMX, MI, MCD.
3. CII, DXI, MDXXXIX, MMMXXX, MMDCCCLVIII, CCCL
4. CCCXXXIII, $\overline{X}$, $\overline{XC}$, $\overline{V}$M, $\overline{V}$CMLXXVII, $\overline{XXV}$MMXXVII, $\overline{XL}$CDXLIV.
5. MDCCCXCIX, MMCCXXII, M$\overline{V}$DV, M$\overline{X}$DCIV. $\overline{V}$MMMDCCCLXXXVIII.

Express the following common numbers in Roman Numerals:

6. 202, 47, 91, 80, 20, 77, 101, 10, 111, 606.
7. 437, 908, 899, 763, 497, 829, 827, 999, 888.
8. 2233, 3232, 3333, 4321, 1234, 5678, 8765.
9. 9999, 25671, 891347, 912342, 16713.
10. 191919, 29134, 23476, 912345, 1678942, 3456713.

RECAPITULATION AND EXAMINATION QUESTIONS.

1. *Question.* What is Arithmetic?
Answer. Arithmetic is the study of numbers.

2. *Q.* What are numbers?
A. Numbers are expressions or characters that represent one or more things of the same kind.

3. *Q.* What is *unity* or the *unit* of a number?
A. Unity or the unit of a number is one of the equal things that the number expresses.

4. *Q.* In the number 19 horses what is the unit?
A. One horse.

5. *Q.* What is the unit in the number 26 shillings?
A. One shilling.

6. *Q.* What is the unit in 16 days? 19 cows? 107 beans? 3 farthings? 198 lbs.? 607? 43 bushels? 293? 769 pens?

7. *Q.* How many modes are there of writing numbers, and what are they?
A. There are two methods of expressing numbers, 1st, by words, and 2d, by characters.

8. *Q.* What is Notation?
A. Notation is the expression of numbers by characters.

9. *Q.* What is Numeration?
A. Numeration is the reading of numbers expressed by characters.

10. *Q.* What different characters are used for the expression of numbers?
A. Numbers are expressed either by letters or by figures.

11. *Q.* What is Roman Notation?
   A. Roman Notation is the art of expressing numbers by certain letters of the alphabet.

12. *Q.* What are the seven numeral letters employed in Roman Notation, and what are their values?
   A. I = 1, V = 5, X = 10, L = 50, C = 100, D = 500, and M = 1000.

13. *Q.* How many times may each of these letters, except V, L, and D, be repeated; and when thus repeated, what do they mean?
   A. No letter can be repeated more than *three times;* and when a letter is thus repeated, its value is repeated.

14. *Q.* When a letter of a lower value is written *before* one of a higher, what does the notation imply?
   A. When a letter of a lower value is written before one of a higher, its value is to be subtracted from that of the higher.

15. *Q.* When a letter of a lower value is written after one of a higher, what does the notation imply?
   A. When a letter or repetition of letters of a lower value comes after a letter of a higher value, the notation implies that their values are to be added.

16. *Q.* What effect has a bar or a dash over a letter or combination of letters?
   A. A bar or a dash written over a letter or combination of letters, increases the value a thousand fold.

17. *Q.* What letters are never written before others?
   A. V, L, D, are never written before letters of a higher value.

18. *Q.* What letter is never written with a bar over it, and why?
   A. I; because we have already an expression for 1000, viz. M.

19. *Q.* What are the figures used in Arabic Notation?
   A. The figures employed in Arabic or Common Notation are 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

20. *Q.* What are the figures 1, 2, 3, 4, 5, 6, 7, 8, 9 called, and why?
   A. The figures 1, 2, 3, 4, 5, 6, 7, 8, 9 are called *significant* figures, because each of them represents one or more individual things.

21. *Q.* By what other name are they also known, and why?
   A. They are also called *digits*, from a Latin word meaning "a finger," because many persons habitually count on the fingers.

22. *Q.* What is the character 0 called, and why?
   A. The character 0 is called naught, nothing, cipher, or zero, because it has no value in itself, and is merely used to give the digits their proper *place*.

23. *Q.* What is meant by the place of a digit?
   A. A digit is said to occupy the first, second, third, fourth, fifth, sixth, &c., *place*, according as it is the last digit to the right hand of the number, last but one, last but two, last but three, last but four, last but five, &c.

24. *Q.* What names are given to the different orders or places beginning at the right-hand side?
A. Units, tens, hundreds, thousands, tens of thousands, hundreds of thousands, millions, tens of millions, hundreds of millions, billions, &c.

25. *Q.* In what ratio do numbers increase in value as we proceed from right to left?
A. In a tenfold ratio, i. e. ten units make one of the order of tens; ten tens, one of the order of hundreds; ten hundreds, one of the order of thousands, &c.

26. *Q.* How does writing ciphers after a number affect its value?
A. Each cipher multiplies the number by 10. Thus 9 = nine, 90 = ninety, 900 = nine hundred, 9000 = 9 thousand, &c.

27. *Q.* What is a period of numbers?
A. A period of numbers is a set or sequence of three digits or ciphers.

28. *Q.* Why are periods used?
A. Periods are used to facilitate the reading of numbers.

29. *Q.* What are the names of the periods beginning with the right-hand or lowest?
A. Units, thousands, millions, billions, trillions, &c.

30. *Q.* What is the rule for reading any large number?
A. 1st. Begin at the right-hand side and point off into periods of three figures each.
2d. Then, commencing at the left-hand side, read each period in succession, unless it be wholly filled with ciphers, in which case pass it by altogether.

31. *Q.* What is the rule for writing numbers?
A. Begin at the left hand and fill each period in succession with the digits or ciphers that belong to it.

SIMPLE ADDITION.

21. Addition teaches us how to find the sum of two or more numbers.

MENTAL EXERCISES.

1. Count aloud up to one hundred.
2. How many do 1 and 1 make? 2 and 1? 3 and 1? 4 and 1? 5 and 1? 6 and 1? &c.
3. How many do 2 and 2 make? 4 and 2? 6 and 2? 8 and 2? 10 and 2? &c.
4. How many do 1 and 3 make? 4 and 3? 7 and 3? 10 and 3?
5. How many do 1 and 4 make? 5 and 4? 9 and 4? 13 and 4? &c.
6. How many do 1 and 5 make? 6 and 5? 11 and 5? 16 and 5? &c.

7. How many do 1 and 6 make? 7 and 6? 13 and 6? 19 and 6? &c.
8. How many do 1 and 7 make? 8 and 7? 15 and 7? &c.
9. How many do 1 and 8 make? 9 and 8? 17 and 8? &c.
10. *How many do 1 and 9 make? 10 and 9? 19 and 9? &c.
11. How many are $7 and $8 and $5 and $9 and $2?
12. How many are 6 apples and 2 apples and 3 apples and 5 apples and 7 apples and 9 apples?
13. How many are 6 pegs and 8 pegs and 7 pegs and 4 pegs and 3 pegs and 2 pegs and 9 pegs and 5 pegs and 1 peg?
14. 16 and 9 are how many? 26 and 9? 37 and 9? 42 and 9? 71 and 9?
15. How many are 15 and 8? 23 and 8? 35 and 8? 39 and 8? 17 and 8?
16. How many are 6 and 7? 21 and 7 32 and 7? 43 and 7? 54 and 7?
17. How many are 5 and 6? 7 and 9? 18 and 4? 23 and 8? 37 and 8?
18. How many are 11 and 9? 13 and 8? 15 and 7? 17 and 6? 18 and 5?
19. How many are 9 and 5? 13 and 7? 27 and 9? 82 and 7? 93 and 9?
20. How many are 7 and 2? 9 and 8? 17 and 6? 23 and 9? 32 and 7? 9 and 9?
21. Jane paid 7 cents for apples, 16 cents for cakes, 9 cents for nuts, and 8 cents for candy; how much did she pay for the whole?
22. William gave 23 cents for a ball, 9 cents for a top, 5 cents for a cord, 8 cents for a bat, and 9 cents for pencils; what did he pay for the whole?
23. A farmer receives $67 for flour, $9 for potatoes, $7 for butter, $6 for turnips, $9 for straw, $8 for cheese, and $9 for poultry; how much does he get for the whole?
24. A gentleman bought 27 books in January, 8 in February, 9 in March, 6 in April, 9 in May, 8 in June, 9 in July, 7 in August, 8 in September, 9 in October, 8 in November, and 7 in December; how many did he buy in all?
25. Fanny worked 7 problems on Monday, 9 on Tuesday, 8 on Wednesday, 9 on Thursday, 6 on Friday, and 8 on Saturday; how many did she work in the entire week?

22. The numbers to be added together are called the *addends*, and the result of the addition is called the *sum*.

---

* The pupil should be continued at this exercise until he can count by 2's, 3's, 4's, 5's, &c., with as much facility as he can by 1's. For example, beginning at any number, say 17, he should be able to count rapidly by *twos*; thus, 17, 19, 21, 23, 25, &c.; or by *threes*; thus, 17, 20, 23, 26, 29, 32, &c.; or by *sevens*; thus, 17, 24, 31, 38, 45, &c. In fact, he cannot add with ease and comfort until he has been well drilled in some such exercise.

23. The sign +, called *plus*, written between two numbers, indicates that they are to be added together. Thus 2 + 6, read 2 *plus* 6, means that 2 and 6 are to be added together; 6 + 9 + 5 + 7 means that 6, 9, 5, and 7 are to be added together.

24. The sign =, called the sign of equality, written between two quantities or expressions, indicates that they are equal to one another. Thus 6 + 7 + 9 = 22, read 6 *plus* 7 *plus* 9 *equals* 22, means that the sum of 6, 7, and 9 is equal to 22.

RULE FOR SIMPLE ADDITION.

25. *Write the addends under one another so that units come under units, tens under tens, hundreds under hundreds, &c.*

Add up each column separately, beginning at the right-hand side. If the sum of the digits in any column does not exceed 9, set it down under that column; but if the sum of the digits in any column does exceed 9, set down only the right-hand figure of the sum under that column, and carry the other or others to the next.

Proof.—First Method.—*Begin at the top and add the columns downwards. The sum should be the same as that obtained before.*

Second Method.—*Cut off the top addend; add the others together, and to their sum add the top addend. The entire sum thus obtained should be the same as that found by the rule.*

Example 1.—Add together 4278, 1610, and 3001.

OPERATION.
4278
1610
3001
——
8889

Here we set down the numbers according to the rule, and, adding up each column, we find the sum in each case does not exceed 9, and accordingly set it down under its appropriate column.

EXERCISE 5.

(1)	(2)	(3)	(4)	(5)	(6)
123	1231	1111	1000	11000	906040
201	3412	2222	213	1200	90340
222	1111	3100	4010	600	1306
333	3031	3103	1201	26104	9

7. How many are 713 + 80 + 3?
8. How many are 12100 + 2210 + 1001 + 421 + 10002?
9. How many are 1020 + 304 + 1111 + 3212?
10. How many are 222 + 1111 + 3333 + 1212 + 90000?
11. How many are 60004 + 8000 + 741 + 21000?
12. Add together twenty-three, four hundred and sixteen, and three thousand and sixty.

EXAMPLE 2.—Find the sum of 478, 693, and 492.

OPERATION.

```
 478
 693
 492
----
1663
```

Here we set down the numbers according to the rule, and, adding up the first column, we find its sum to be 13, of which we set down the 3 under the first column and carry the 1 to the second. The sum of the second column, with the one carried, is 26, of which we set down the right-hand figure, 6, under the column added, and carry the 2 to the next. The third column, added, amounts to 16, which we set down in full.

EXAMPLE 3.—Add together 7149, 7132, 614, 9137, 1234, and 79813.

OPERATION.

```
  7149
  7132
   614
  9137
  1234
 79813
------
105079
```

Here the first column amounts to 29, of which we set down the right-hand figure 9, and carry the 2 to the second column. The sum of the second column, with the 2 carried, is 17, of which we set down the 7 and carry the 1 to the third column. The sum of the third column, with the 1 carried, is 20, of which we set down the right-hand figure, 0, and carry the 2 to the fourth column. The sum of the fourth column, with the 2 carried, is 35, of which we set down 5 or right-hand figure and carry the 3 to the fifth column. The sum of the fifth or last column, with the 3 carried, is 10, which we set down in full.

EXERCISE 6.

(1)	(2)	(3)	(4)
12345	2233	718645	91600
67134	4567	191371	7149
91317	8912	234716	86004
19134	3456	918130	19136

(5)	(6)	(7)	(8)
71461	11111	78912	13456
9147	2222	3456	7
90061	333	78	987
876	44444	9	29
4713	5555	98765	98613

(9)	(10)	(11)	(12)
80476	123456	34567	723
9007	789123	8000	674
986147	456789	69	1674
91067	123456	470000	19006
86	789123	109687	1916
4071	456789	48001	986986
937	987654	290	97979

(13)	(14)	(15)	(16)
987654	8000700	8147137	987654
32109	600090	913714	137867
8765	1129000	9100070	149167
432	47896	8000000	891371
10	8104906	667755	919198
9	23427	44332	171296
87	9867	3355778	147867
654	999999	986754	182371
3210	88888	71347	929292
98765	710	981675	292929
432109	9134761	19198	777777

17. Find the sum of 1247 + 91679 + 27 + 1987 + 1800 + 1796.

18. Find the sum of 13147 + 9 + 61946 + [illegible] + 1410 + 954 + 1346 + 16137 + 209 + 76649 + 8 + 967.

19. How many are 6 + 27 + 93 + 47 + 679 + 496 + 9999?

20. How many are 12 + 21 + 679 + 976 + 769 + 9198 + 4617 + 9860?

21. Add together four hundred and sixty, seven thousand and nineteen, four thousand eight hundred and fifty, nine thousand and twenty-six, seven thousand nine hundred and ninety-nine, one thousand four hundred, six thousand and twenty-one, five thousand and eighty-seven, and four thousand five hundred and sixty-seven.

22. Add together twenty-seven thousand and sixteen, eight thousand and seven, sixty thousand four hundred and twenty-five, eighty-four thousand six hundred and eleven, nineteen thousand and nineteen, fifty-five thonsand seven hundred, and ninety thousand seven hundred and four.

23. Add together sixty-seven thousand and nine, forty-nine thousand six hundred and eighty-six, five hundred and twenty-five thousand and sixteen, three thousand and eleven, eighty-five thousand seven hundred and twenty-seven, and sixteen thousand and seven.

24. Add together two hundred and seven thousand six hundred and nine, eleven millions and sixteen, five millions four hundred thousand seven hundred and twenty, sixty-six millions two hundred and twenty-nine thousand and eighty-seven, nine hundred and eighty-seven millions six hundred and seventeen, and five thousand seven hundred and thirty-five.

25. An apple woman sold forty-seven apples on Monday, eighty-nine on Tuesday, two hundred and seventeen on Wednesday, one hundred and four on Thursday, one hundred and twenty on Friday, and two hundred and eighty-seven on Saturday: how many did she sell during the week?

26. A farmer sent five loads of oats to market. The first load contained 63 bushels, the second 58 bushels, the third 79 bushels, the fourth 57 bushels, and the fifth 63 bushels. How many bushels were there in the five loads?

27. The imports of the six principal ports of Canada for 1855 were as follows: Montreal, $13528156; Toronto, $6420224; Quebec, $4566376; Hamilton, $3545508; Kingston, $849302; and Port Stanley, $524264. What was the total value of the imports at these six places?

28. During 1848 there were exported from Canada 2248016 bushels of wheat; in 1849 there were exported 3645320 bushels; in 1850, 4547224 bushels; in 1851, 4275896 bushels; and in 1852, 5496718 bushels. How many bushels of wheat were exported from Canada during the five years ending 1852?

29. A farmer has four fields of turnips; the first yields him 743 bushels, the second 1147 bushels, the third 890 bushels, and the fourth 1279 bushels; how many bushels of turnips did he obtain from the four fields?

60. At one of the large boot and shoe factories in Montreal, the work turned out during a week was as follows: Monday, 1497 pairs of shoes, Tuesday, 1509 pairs, Wednesday, 1164 pairs, Thursday, 1447 pairs, Friday, 1523 pairs, and Saturday, 1498 pairs; how many pairs of shoes were made in this factory during the week?

61. When will a person who was born in the year 1793 be 84 years of age?

(32)	(33)	(34)	(35)	(36)
12	987	27145	753	16
34	613	91913	197	61
56	479	16719	531	81
78	813	91871	975	47
91	271	49181	319	29
23	986	37162	864	87
45	129	34567	208	46
67	333	89134	642	98
89	400	56789	186	63
98	916	12345	421	42
76	713	67890	987	17
54	934	12345	565	93
32	716	67891	321	82
10	291	23456	123	54
98	816	78912	456	70
76	999	34567	789	62
54	816	89123	808	16
32	554	45678	707	17
11	735	91234	606	91
23	613	56789	404	87
45	421	71642	505	63
66	916	97531	660	22
77	818	24680	770	71
88	397	90406	880	33
99	491	71430	990	42
35	378	61311	169	71
79	613	44433	178	93
24	491	91671	144	45
68	351	82916	916	67
90	673	57137	723	16
81	916	91346	444	49
85	814	13471	718	98
71	716	91399	999	80
80	537	12916	800	73
98	981	71307	437	34

NOTE.—These long columns are designed to practise the pupil upon rapidity in adding. He is not to spell them up by saying 8 and 6 make 14, 14 and 1 make 15, 15 and 5 make 20, &c., but to read them up by simply touching each digit with his pencil and naming the sum; thus, 8..14..15..20..21..29..33..42, &c.

RECAPITULATION AND EXAMINATION QUESTIONS

1. *Question.* What is Addition?
Answer. Addition is the process of finding the sum of two or more numbers.

2. *Q.* What are the numbers to be added called?
A. The numbers to be added are called *addends.*

3. *Q.* What is the result of the addition called?
A. The result of the addition is called the *sum.*

4. *Q.* When two or more numbers are to be added together, what is the first thing to be done?
A. When numbers are to be added, we first write them under one another so that units come under units, tens under tens, &c.

5. *Q.* What is the next part of the rule for addition?
A. We next draw a line under the addends to separate them from the sum.

6. *Q.* What is the next thing done?
A. We next add up the units' column, set down the right-hand figure of the sum under the column of units, and carry the other figure or figures to the tens' column.

7. *Q.* What is next done?
A. We next add up the tens' column, together with what we carried from the units, write down the right-hand figure of the sum under the column of tens, and carry the other figure or figures to the hundreds' column.

8. *Q.* Why do we set down the right-hand figure under the column added, and carry the other figures to the next column to the left?
A. Because, when we have added up any column, the right-hand figure expresses units of that order, and the other figure or figures so many tens, and ten of any one order make one of the next higher.

. *Q.* How is the sign of addition written, and what is its name?
A. The sign of addition is written thus +, and is called *plus.*

.. *Q.* What is the sign = called, and what does it mean?
A. The sign = is called the sign of equality, and it means that the quantities between which it is written are equal to one another.

.. *Q.* Explain what is meant by the following expression:

$$17 + 200 + 40 + 3 = 167 + 9 + 11 + 33 + 40.$$

A. It means that the sum of 17, 200, 40, and 3, is equal to the sum of 167, 9, 11, 33, and 40.

Q. How may addition be proved?
A. We may prove addition by adding the columns over again from the top downward.

Q. In what other way may addition be proved?
A. We may prove addition by cutting off the top addend, adding the others together, and then to their sum adding the top addend. The entire sum thus obtained should agree with that found by the rule.

SIMPLE SUBTRACTION.

26. Subtraction is the process of finding the difference between two numbers.

27. The sign —, called *minus*, written between two numbers, indicates that the one following the sign is to be subtracted from the one preceding it. Thus 16 — 9, read 16 *minus* 9, means that 9 is to be subtracted from 16.

28. The number to be subtracted is called the *subtrahend*, and the other number the *minuend*. What is left after the subtraction, is called the *remainder* or *difference*.

MENTAL EXERCISES.

1. From 100 take 1, and how many remain? From 99 take 1, and how many remain? From 98 take 1, and how many remain? &c.
2. From 100 take 2, and how many remain? From 98 take 2, and how many remain? From 96 take 2, and how many remain? &c.
3. From 100 take 3, and how many remain? From 97 take 3, and how many remain? &c.
4. From 100 take 4, and how many remain? From 96 take 4, and how many remain? &c.
5. How many are 100 — 5? 95 — 5? 90 — 5? 85 — 5? &c.
6. How many are 100 — 6? 94 — 6? 88 — 6? &c.
7. How many are 100 — 7? 93 — 7? &c.
8. How many are 100 — 8? 92 — 8? &c.
9. How many are 100 — 9? 91 — 9? &c.
10. How many are 87 — 2 — 3 — 4 — 5 — 9 — 8 — 7 — 6 — 4 — 5?
11. How many are 80 — 9 — 6 — 7 — 1 — 2 — 4 — 5 — 6 — 7 — 4?
12. How many are 9 + 3 + 4 + 9 + 7 + 6 — 7 — 4 — 3 — 7 — 9 — 1 — 4?
13. How many are 6 + 1 + 5 + 2 + 1 + 3 + 7 — 2 — 1 — 3 — 7 + 9 + 8 — 9 — 4?
14. How many are 6 + 7 + 4 — 9 — 8 + 2 + 9 — 8 + 6 + 7 — 3 — 2 + 7 — 9?
15. How many are 19 + 6 + 7 — 3 — 4 — 7 + 8 — 6 — 2 + 1 + 9 — 3 — 4 — 2 + 7 — 8?
16. Fannie has 17 apples and Martha 9; how many more apples has Fannie than Martha?
17. Emma worked 7 questions in arithmetic each day of the week, while Maggie worked 8 questions on Monday, 5 on Tuesday, 10 on Wednesday, 7 on Thursday, 3 on Friday, and 9 on Saturday: how many more questions did Emma solve during the week than Maggie?

18. Thomas went to market with 27 dollars. He paid 7 dollars for poultry, 3 dollars for vegetables, 4 dollars for fruit, and 1 dollar for fish. How much money had he left?

19. George has 63 cents, and he gives 9 cents to Florence and 8 cents to Charles. How many has he remaining?

20. A farmer had a flock of 34 sheep. He lost 7, sold 9, gave away 8, killed 4, and kept the rest. How many did he keep?

21. Lizzie has 37 cents. She spends 4 cents for a pencil, 7 cents for paper, 3 cents for ink, 9 cents for a copy-book, and the remainder for a slate. How much did she give for the slate?

22. A man has 35 cords of wood to saw, and wishes to finish it in 5 weeks. He saws 3 cords the first week, 5 cords the second, 8 cords the third, and 9 cords the fourth. How many cords remain for him to saw in the fifth week?

RULE FOR SUBTRACTION.

29. *Write the subtrahend under the minuend so that units come under units, tens under tens, &c.*

Commence at the right-hand side, and subtract each figure of the subtrahend from the corresponding figure of the minuend, and set down the remainder in the same column. If in any case a figure of the subtrahend is greater than the corresponding figure of the minuend, increase the latter by 10, *and then carry one to the next figure of the subtrahend.*

PROOF.—*Add the* DIFFERENCE *to the* SUBTRAHEND, *and the sum should equal the minuend. Or,*

Subtract the DIFFERENCE *from the* MINUEND, *and the remainder should equal the subtrahend.*

EXAMPLE 1.—Find the difference between 167947 and 32717.

OPERATION.

```
167947 Minuend.
 32717 Subtrahend.
------
135230 Difference.
------
167947 Proof.
```

Here setting down the given numbers according to the rule, we subtract each digit of the subtrahend from the one above it, and set down the remainder in the same column.

Then to prove our work, we add the difference to the subtrahend; and since the sum thus obtained is equal to the minuend, we conclude the work is correct.

EXERCISE 7.

(1)	(2)	(3)	(4)
69147934	4613598	619145	6191890
18003932	1203068	19143	6190840
(5)	(6)	(7)	(8)
9876543	129147	914718	898906
543210	20034	212206	287103

9. *From* eight hundred and seven millions nine hundred and sixty-five thousand seven hundred and seventy-eight, *take* six hundred and three millions eight hundred and sixty-one thousand seven hundred and twenty-five.

10. *From* five millions seven hundred and four thousand nine hundred and eighty, *take* five millions three hundred and four thousand six hundred and forty.

EXAMPLE 2.—From 723 take 571.

OPERATION.
723
571
——
152

Here in the second column, we cannot take 7 from 2, so we increase the 2 by 10 and thus make it 12. Then we say 7 from 12 and 5 remain. Next we add 1 to the 5 in the subtrahend, and say 6 from 7 and 1 remains.

EXAMPLE 3.—From 71006 take 9867.

OPERATION.
71006
9867
——
61139

Here we say 7 from 6 we cannot, but, increasing the 6 by 10, we say 7 from 16 and 9 remain. The 7 (i. e. 6 with 1 added) from 0 we cannot, but, increasing the 0 by 10, we have 7 from 10 and 3 remain. Then 9 from 0 we cannot, but 9 from 10 (i. e. 0 and 10) and 1 remains. 10 from 1 we cannot, but 10 from 11 and 1 remains. Lastly, 1 from 7 and 6 remain.

EXERCISE 8.

(1)	(2)	(3)	(4)
676643	816427	16134	291860
12571	13518	5317	119137
(5)	(6)	(7)	(8)
8167140	1910042	810006	9000470
914067	191008	79867	916439

(9)	(10)	(11)	(12)
80000007	8043007	960007008	600400070
9149136	3429168	9989986	19140607

13. Find the difference between 70460 and 9086.
14. Find the difference between 800000 and 98647.
15. Find the difference between 407080 and 998.
16. *From* seventy-six millions and eight *take* eleven millions nine hundred and seventy-eight thousand five hundred and twenty-nine.
17. *From* four millions seventy thousand and ninety *take* six hundred and eighty thousand seven hundred and four.
18. *From* twenty-seven millions forty-three thousand and six *take* twenty millions seven hundred thousand and eighty.
19. A farmer who had nine hundred and sixty bushels of potatoes sold five hundred and seventy-six bushels. How many bushels had he remaining?
20. From a library containing forty-seven thousand and ninety volumes, twenty-eight thousand seven hundred and ninety-six were sold. How many volumes remained?
21. The entire population of British North America is about 4260000. The population of Canada is 3000000; what is the population of the remaining provinces of British North America?
22. Henry has 276 marbles; he gives 17 to Tom, 61 to Willie, 63 to Robert, and 24 to John; how many has he left?
23. A merchant paid 12 dollars for figs, 17 dollars for raisins, 29 dollars for spice, 164 dollars for sugar, 716 dollars for tea, 690 dollars for coffee and wine. He carried with him 2700 dollars; after paying for all these goods how much had he left?
24. Three merchants began business with a capital of $1793; the first had $714, the second had $417, what did the third have?
25. A man paid $17896 for a steamboat and afterwards sold it for $14070; how much did he lose by his bargain?
26. If you add 769 to 861 and then subtract 708 from the sum, what will remain?
27. What sum added to 6159 will make twenty-seven millions and seventeen?
28. The population of Toronto is 50000, Montreal 80000, Hamilton 25000, Quebec 45000, London 15000, Ottawa 10000, and Kingston 15000; how much less is the entire population of these seven cities than that of New York, which is 600000?
29. What five numbers each greater than 100 together make 743?
30. Resolve the number 4146 into any four addends none of which shall be less than 120.

31. The minuend is 74967 and the remainder 6943, what is the subtrahend?

32. The subtrahend is 64009 and the remainder 7143, what is the minuend?

33. What sum will leave 1727 when 917 is taken from it?

34. Four loads of wheat together weighed 9169 lbs. The first weighed 2007 lbs., the second 1963 lbs., and the third 2614 lbs.: what was the weight of the fourth?

35. What two numbers, neither of them less than 740, will together make 9137?

(36)	(37)	(38)	(39)
1789437	913471	167142	987671
—12371	—61312	—9347	—81432
—29867	—91314	—91671	—13427
—14371	—6713	—9181	—90000
—3198	—9147	—76	—6714
1729630			

Note.—Add up these numbers and subtract at the same time. Thus in question 36 proceed as follows: 8 and 1 make 9 and 7 make 16 and 1 make 17; 17 from 7 we can't, borrow 10, then 17 from 17 and 0 remains; 1 (carried) and 9 make 10 and 7 make 17 and 6 make 23 and 7 make 30; 30 from 3 we can't, borrow 3 of the next order, then 30 from 33 and 3 remains; 3 (carried) and 1 make 4 and 3 make 7 and 8 make 15; and so on.

(40)	(41)	(42)	(43)
194362	734713	3427	314267
—7143	—23142	—67142	—267
—21713	— 61714	98134	—98
61429	91317	278	417132
—21347	—23916	—6179	—98067
205588			

Note.—In these questions, bearing in mind that the numbers preceded by the sign — are to be taken in a subtractive sense, and those without any sign in an additive sense, proceed as follows: $-7+9=2$ and $-3=-1$ and $-3=-4$ and $+2=-2$ and 2 from $10=8$ to set down; next one borrowed, and therefore -1, and $-4=-5$ and $+2=-3$ and $-1=-4$ and $-4=-8$ and $+6=-2$ and 2 from $10=8$ to set down; next -1 (1 borrowed) and $-3=-4$ and $+4=0$ and $-7=-7$ and $-1=-8$ and $+3=-5$ and 5 from $10=5$ to set down, &c.

(44)

2387674327198716714391008647132987679
 191898109123491816127189161713429181

(45)

9120001200134100210001304132171329816
8079113406914718913291346129131421986

(46)

1234567890001230004560007890000043007
1119189167452371342912345678911254569

(47)

8091002000300040000500000678009720003
914867198671491813471986714986914271

(48)

98181000000000006000700040098167007100
1234567890120071901300410910007180981

NOTE.—Questions 44–48 are intended to exercise the pupil in rapidity in subtraction. Simply touching the digit in the subtrahend and that in the minuend, he should at once name the figure to be set down. Thus, 1..9..8; 8..17..9; 2..6..4; 9..17..8; 3..8..5, &c.

RECAPITULATION AND EXAMINATION QUESTIONS.

1. *Question.* What is Subtraction?
   Answer. Subtraction is the process of finding the difference between two numbers.
2. *Q.* What is the number to be subtracted called?
   A. The number to be subtracted is called the *subtrahend.*
3. *Q.* What is the number from which the subtrahend is to be taken called?
   A. The number from which the subtrahend is to be taken is called the *minuend.*
4. *Q.* What is the number resulting from the subtraction called?
   A. The number resulting from the subtraction is called the *difference or remainder.*
5. *Q.* How is the sign of subtraction written, and what is it named?
   A. The sign of subtraction is written thus —, and is called *minus.*
6. *Q.* What is the first part of the rule for subtraction?
   A. We are first directed to write the subtrahend under the minuend so that units come under units, tens under tens, &c.
7. *Q.* What is the second part of the rule for subtraction?
   A. After drawing a line below the subtrahend to separate it from the remainder, we subtract the right-hand figure from that over it and set down the remainder in the same column; then we subtract the tens' figure of the subtrahend from the tens' figure of the minuend, next the hundreds; and so on.
8. *Q.* When any figure in the subtrahend is greater than the figure of the minuend directly over it, how do we proceed?
   A. When the subtrahend figure is greater than the corresponding minuend figure, we increase the latter by 10 and then add one to the next subtrahend figure to the left.

9. Q. How may subtraction be proved?

A. We may prove subtraction by adding together the *remainder* and the *subtrahend*: the sum should be equal to the minuend.

10. Q. How may subtraction be proved by subtraction?

A. By subtracting the remainder from the minuend; the result should equal the subtrahend.

SIMPLE MULTIPLICATION.

30. Multiplication is a short process of taking one number as many times as there are units in another.

31. The number to be multiplied is called the *multiplicand.*

32. The number by which we multiply is called the *multiplier.*

33. The number resulting from the multiplication is called the *product.*

34. The multiplier and the multiplicand are called the *factors* of the product.

35. An integer or integral number is a whole number. Integers are either prime or composite.

36. A prime number is a number which cannot be exactly divided by any integral number except unity and itself. Thus 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, &c., are prime numbers.

37. A composite number is the product of two or more integral numbers, neither of which is unity. Thus 4, 6, 8, 9, 12, 14, 15, 16, 18, 20, 21, 24, 25, &c., are composite numbers.

38. The sign ×, called the sign of multiplication, written between two numbers, indicates that they are to be multiplied together. Thus 16 × 7, read 16 *multiplied by* 7, means that 16 is to be multiplied by 7.

MULTIPLICATION TABLE.

Twice	3 times	4 times	5 times	6 times	7 times
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6	1 are 7
2 — 4	2 — 6	2 — 8	2 — 10	2 — 12	2 — 14
3 — 6	3 — 9	3 — 12	3 — 15	3 — 18	3 — 21
4 — 8	4 — 12	4 — 16	4 — 20	4 — 24	4 — 28
5 — 10	5 — 15	5 — 20	5 — 25	5 — 30	5 — 35
6 — 12	6 — 18	6 — 24	6 — 30	6 — 36	6 — 42
7 — 14	7 — 21	7 — 28	7 — 35	7 — 42	7 — 49
8 — 16	8 — 24	8 — 32	8 — 40	8 — 48	8 — 56
9 — 18	9 — 27	9 — 36	9 — 45	9 — 54	9 — 63
10 — 20	10 — 30	10 — 40	10 — 50	10 — 60	10 — 70
11 — 22	11 — 33	11 — 44	11 — 55	11 — 66	11 — 77
12 — 24	12 — 36	12 — 48	12 — 60	12 — 72	12 — 84

8 times	9 times	10 times	11 times	12 times
1 are 8	1 are 9	1 are 10	1 are 11	1 are 12
2 — 16	2 — 18	2 — 20	2 — 22	2 — 24
3 — 24	3 — 27	3 — 30	3 — 33	3 — 36
4 — 32	4 — 36	4 — 40	4 — 44	4 — 48
5 — 40	5 — 45	5 — 50	5 — 55	5 — 60
6 — 48	6 — 54	6 — 60	6 — 66	6 — 72
7 — 56	7 — 63	7 — 70	7 — 77	7 — 84
8 — 64	8 — 72	8 — 80	8 — 88	8 — 96
9 — 72	9 — 81	9 — 90	9 — 99	9 —108
10 — 80	10 — 90	10 —100	10 —110	10 —120
11 — 88	11 — 99	11 —110	11 —121	11 —132
12 — 96	12 —108	12 —120	12 —132	12 —144

MENTAL EXERCISES.

1. How many are twice 2? twice 3? twice 4? twice 5? twice 6? &c.
2. How many are 3 times 2? 3 times 3? 3 times 4? &c.
3. How many are 4 times 2? 4 times 3? 4 times 4? &c.
4. How many are 5 times 2? 5 times 3? 5 times 4? &c.
5. How many are 6 times 2? 6 times 3? 6 times 4? &c.
6. How many are 7 times 2? 7 times 3? 7 times 4? &c.
7. How many are 8 times 2? 8 times 3? 8 times 4? &c.
8. How many are 9 times 2? 9 times 3? 9 times 4? &c.

9. How many are 10 times 2? 10 times 3? 10 times 4? &c.
10. How many are 11 times 2? 11 times 3? 11 times 4? &c.
11. How many are 12 times 2? 12 times 3? 12 times 4? &c.
12. How many are 3 times 7? 7 times 3? How many 7's in 21? How many 3's in 21?
13. How many are 8 times 9? 9 times 8? How many 9's in 72? How many 8's in 72?
14. How many are 6 times 7? 7 times 6? How many 6's in 42? How many 7's in 42?
15. How many are 8 times 8? How many 8's are there in 64?
16. How many are 12 times 9? 9 times 12? How many 12's in 108? How many 9's in 108?
17. How many are 11 times 11? How many 11's in 121?
18. How many are 8 times 6? 6 times 8? How many 8's in 48? How many 6's in 48?
19. How many are 9 times 9? How many 9's in 81?
20. How many are 7 times 8? 8 times 7? How many 8's in 56? How many 7's in 56?

NOTE.—The teacher should continue this exercise until his pupils are thoroughly familiar with the multiplication table.

21. What are the factors of 4? (*Ans.* 2 × 2.) What are the factors of 8? (*Ans.* 4 × 2 or 2 × 2 × 2.) What are the factors of 48? (*Ans.* 8 × 6, or 12 × 4, or 4 × 4 × 3, or 2 × 2 × 4 × 3, or 2 × 2 × 2 × 2 × 3, or 16 × 3, or 24 × 2.)
22. What are the factors of 18? of 20? of 24? of 32? of 36? of 81? of 144?
23. What are the factors of 72? of 84? of 56? of 39? of 108? of 121?
24. What are the factors of 15? of 35? of 42? of 27? of 88? of 100? of 64?

RULE FOR MULTIPLICATION.

39. When the multiplier does not exceed 12.

Place the multiplier under the right hand figure of the multiplicand, and draw a horizontal line beneath.

Begin at the right-hand side, and multiply each figure of the multiplicand by the multiplier, set down the right-hand figure of the partial product under that figure of the multiplicand which produced it, and carry the remaining figure or figures to the next partial product.

EXAMPLE.—Multiply 71497 by 12.

OPERATION.

Multiplicand	71497
Multiplier	12
Product	857964

Here 12 times 7 are 84, and we set down 4 and carry 8; 12 times 9 are 108 and 8 carried are 116, of which we set down the 6 and carry the 11; 12 times 4 are 48 and 11 carried make 59, of which we set down the 9 and carry the 5, &c.

EXERCISE 9.

(1)	(2)	(3)	(4)
71491371	91818947	91134719	67143917
2	3	4	5
(5)	(6)	(7)	(8)
918765421	879165498	12357986	987165498
6	7	8	9
(9)	(10)	(11)	(12)
1671491345	7861491391	4291498671	78674918
10	11	12	8

13. What is the product of 791876 × 3? × 2? × 4? × 12?
14. What is the product of 818619847 × 7? × 8? × 9? × 11?
15. What is the product of 6179 × 3? × 5? × 7? × 9? × 12?
16. What is the product of 6987159 × 10? × 7? × 8? × 6?
17. Multiply 714719 by 12.
18. Multiply 1913476 by 9.
19. How many are 8 times 76598?
20. How many are eleven *times* four hundred millions seven thousand and ninety six?
21. What is the product of 714 × 7? × 11? × 6? × 5? × 12?

---

40. When the multiplier is a composite number, none of whose factors are greater than 12:—

RULE.

Multiply the given multiplicand by any one of the factors of the multiplier; then multiply the product thus obtained by a second factor of the multiplier, and this second product by the third factor if there be one; and so on until all the factors of the multiplier are used.

Example.—Multiply 671908 by 56.

OPERATION.

671908
8
———
5375264
7
———
37626848

Here the multiplier is 56, of which the factors are 8 × 7, and, according to the rule, we first multiply the given number by one factor, and then the result by the other factor.

Exercise 10.

1. Multiply 719867 by 48.
2. Multiply 916704 by 84.
3. Multiply 714367 by 27.
4. Multiply 161714 by 16.
5. Multiply 71698 by 81.
6. Multiply 81897 by 121.
7. Multiply 716914 by 144.
8. Multiply 167149 by 54.
9. Multiply 191878 by 42.
10. Multiply 891476 by 64.
11. Multiply 918978 by 108.
12. Multiply 765439 by 132
13. How many are seventy-two *times* six hundred and four thousand five hundred and seventy-nine?
14. How many are forty nine *times* eight millions nine hundred and sixty-eight thousand four hundred and seventy-six?
15. What will 49 horses cost at $147 each?
16. What will 63 cows cost at $48 each?
17. What will 987 hogsheads of sugar cost at $80 a hogshead?
18. Suppose a book to contain 412 pages, each containing 42 lines, and that each line contains 56 letters, how many letters are there in the whole book?
19. If an apple woman sells 121 apples a day, how many will she sell in a year, which, omitting the Sundays, contains 313 days?

---

41. When the multiplier exceeds 12, and is not a composite number:—

RULE.

I. *Set the multiplier under the multiplicand so that units come under units and tens under tens, &c.*

II. *Multiply the multiplicand by each figure of the multiplier separately, and set down each partial prod* *thus obtained so that the first figure falls directly under that figure of the multiplier by which it was obtained.*

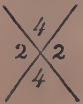
III. *Add the several partial products together as they stand. The sum will be the entire product sought.*

PROOF OF MULTIPLICATION.

42. First Method.—*Multiply the multiplicand by* ONE LESS *than the multiplier, and to the product thus obtained add the multiplicand. The result should be the same as the product obtained by the rule.*

Second Method.—*Cast the 9's out of the multiplicand and set down the remainder, also out of the multiplier and set down the remainder; multiply these two remainders together, and cast the 9's out of their product. The remainder thus found should be the same as that obtained by casting out the 9's from the product of the multiplicand by the multiplier.*

Thus to prove Example 1, we proceed as follows:



7 + 4 + 9 + 6 + 3 = 29, and 29 ÷ 9 gives a remainder, 2, which we write down to the left of a cross, as in the margin.

Next, 2 + 9 = 11, and 11 ÷ 9 gives a remainder, 2, which we write to the right of the cross.

Next, 2 × 2 = 4, and 4 ÷ 9 gives a remainder, 4, which we write above the cross.

Lastly, 2 + 1 + 7 + 3 + 9 + 2 + 7 = 31, and 31 ÷ 9 gives a remainder, 4, which we write beneath the cross. Then, since the number above the cross agrees with that below it, we conclude the work is correct.

Example 1.—Multiply 74963 by 29.

OPERATION.
74963
29
674667
149926
2173927

Here we first multiply the given multiplicand by 9, setting 7, the first figure of the partial product, directly under the 9; next we multiply the given multiplicand by 2, and set down the partial product so that its first figure, 6, falls directly under the 2 by which we are multiplying; lastly, we add the two partial products together just as they stand.

Example 2.—Multiply 714987 by 23004.

OPERATION.

714987
23004
2859948
2144961
1429974
16447560948

Here we first multiply 714987 by 4, setting the first figure of the partial product under the 4; we next multiply by 3, setting the first figure of the second partial product under the 3, and so on; finally, we add the partial products together as they stand.

Note.—Since the multiplicand multiplied by 0 is equal to 0, we pass by the 0's in the multiplier.

Exercise 11.

(1)	(2)	(3)	(4)
7191486	314976	819715	7819164
23	89	698	908
(5)	**(6)**	**(7)**	**(8)**
6540910	7190867	8491791	28700046
8040	8046	91008	90570
(9)	**(10)**	**(11)**	**(12)**
71400600	123456789	91845067	9870643987
900708	98067	900004	9060409

13. What is the product of 71476 × 9187?
14. What is the product of 91476 × 8190?
15. What is the product of 8100070 × 81009?
16. What is the product of 5858857 × 506007?
17. *Multiply* six millions three hundred and seven thousand nine hundred and eighteen *by* twenty thousand seven hundred and ninety.
18. *Multiply* seventy-eight thousand four hundred and eighty six *by* twenty *times* seven thousand and nineteen.
19. *Multiply* seven hundred and forty times nine hundred and seven *by* thirteen *times* two hundred and seventeen.
20. If an acre of wheat yield 29 bushels, how much will 149 acres produce?
21. What will 21 horses cost at $106 each?

22. If 1 hhd. of sugar cost $83, what will 1149 hhds. cost?

23. Montreal is 360 miles distant from Toronto; how many perches are there in this distance, the mile containing 320 perches?

24. If a city contains 97 streets and on an average 304 houses on each street and 17 persons in each house, what will be the entire population of the city?

25. If a library consists of 183 shelves, each shelf containing 53 volumes, and if the average number of pages of reading is 307 to a volume, how many pages of reading does the library contain?

26. If a township contains 23 schools, and each school averages 47 pupils, how many children are there attending school in the township?

(27)
987671813407198767988699756
8

(28)
817614923569871908147634567
11

(29)
130579864213579843212345678
9

(30)
811476193457899986888776654
12

[*] NOTE. These long lines are intended to exercise the pupil in rapid multiplication. He should be required to name the figure to set down with as few intermediate words as possible. Thus, instead of saying 8 times 6 are 48, set down 8 and carry 4; 8 times 5 are 40 and 4 make 44, set down 4 and carry 4, &c., he should be taught to simply touch each figure with his pencil and merely name the digit in the multiplicand, the multiplier, and the figure to be set down, as follows: 8...6...8, 8...5...4, 8...7...0, 8...9...8, &c.

RECAPITULATION AND EXAMINATION QUESTIONS

1. *Question.* What is multiplication?
   Answer. Multiplication is a short method of taking one number as many times as there are units in another.

2. *Q.* What is the number to be multiplied called?
   A. The number to be multiplied is called the *multiplicand.*

3. *Q.* What is the number by which you multiply called?
   A. The number by which we multiply is called the *multiplier*

4. *Q.* What is the result of the multiplication called?
   A. The number resulting from the multiplication is called the *product.*

5. *Q.* What are the factors of a number?
   A. The factors of a number are those numbers which, multiplied together, produce it. Thus the multiplier and multiplicand are the factors of the product.

6. *Q.* What is an integer or integral number?
   A. An integer or integral number is a whole number.

7. *Q.* Of how many kinds are Integers?
   A. Integers are of two kinds, *prime* or *composite.*

8. *Q.* What is a prime number?
   A. A prime number is a number which has no Integral factors except itself and unity.

9. *Q.* What are all the prime numbers less than 100?

10. *Q.* What is a composite number?
    A. A composite number is the product of two or more integral factors neither of which is unity.

11. *Q.* What are all the composite numbers less than 100?

12. *Q.* How is the sign of multiplication written?
    A. The sign of multiplication is written thus, ×.

13. Give the rule for multiplication when the multiplier does not exceed 12. (See Art. 39.)

14. *Q.* In this and the other rules for multiplication, why do we begin multiplying at the *right* hand side?
    A. We begin at the right-hand side in multiplication for the same reason that we begin at the right hand side in addition, i. e. in order to take advantage of the principle of carrying.

15. *Q.* What do you understand by the principle of carrying?
    A. When we have obtained the product of any two digits in multiplication, or the sum of any column in addition, we set down the right-hand figure in that column and *carry* the other figure or figures to the next product or next column, and are thus enabled to do by one process what would otherwise require several.

16. Give the rule for multiplication when the multiplier can be broken up into two or more factors, neither of them greater than 12. (See Art. 40.)

17. Give the rule for multiplication when the multiplier is not composite and is greater than 12. (See Art. 41.)

18. *Q.* In this latter rule, why are you directed to write the right-hand figure of each partial product directly under that figure of the multiplier by which it was obtained?

A. We are thus directed because units multiplied by any order will give that order for product. Thus, the units of the multiplicand, multiplied by the units of the multiplier, give units for product, and we set down the right-hand figure of this partial product under the units; so the units of the multiplicand, multiplied by the tens of the multiplier, give tens for the product, and we therefore commence the partial product under the tens in the multiplier. Similarly, the units of the multiplicand, multiplied by the hundreds of the multiplier, give hundreds for the product, &c.

19. *Q.* How do you multiply by 10, 100, 1000, 10000, &c.?
A. We multiply any number by 10, 100, 1000, &c., by annexing *one*, *two*, *three*, &c., ciphers to the multiplicand.

20. Explain how multiplication may be proved, and illustrate the proof, by casting out the 9's, in examples 1-12 in Exercise 11.

SIMPLE DIVISION.

43. Division teaches the method of finding how many times one number is contained in another.

44. The number to be divided is called the *dividend.*

45. The number by which we divide is called the *divisor.*.

46. The number showing how many times the divisor is contained in the dividend is called the *quotient.*

47. If the divisor is not contained in the dividend an exact number of times, there is left after the division a number called a *remainder.*

NOTE.—The remainder is of the same name as the dividend, because it is a part of it and must always be less than the divisor.

48. The sign ÷, called the sign of division, written between the numbers, indicates that the one preceding the sign is to be divided by that following it. Thus 16 ÷ 4, read 16 *divided* by 4, means that 16 is to be divided by 4.

NOTE.—The division of one number by another is also indicated by writing one above the other, or by writing a colon between them. Thus 27 ÷ 4, or $\frac{27}{4}$, or 27 : 4, each means 27 divided by 4.

49. When the divisor does not exceed 12, the rule is called *short division;* but when the divisor is greater than 12, it is called *long division.*

MENTAL EXERCISES.

1. How many times is 2 contained in 8? in 10? in 18? in 11? in 23?
2. How many times is 3 contained in 9? in 15? in 27? in 33? in 17?
3. How many times is 4 contained in 20? in 28? in 44? in 36? in 19?
4. How many times is 5 contained in 35? in 10? in 50? in 25? in 28?
5. How many times is 6 contained in 18? in 42? in 54? in 36? in 40?
6. How many times is 7 contained in 35? in 7? in 21? in 63? in 25?
7. How many times is 8 contained in 24? in 72? in 96? in 40? in 57?
8. How many times is 9 contained in 81? in 45? in 18? in 72? in 60?
9. How many times is 10 contained in 10? in 40? in 100? in 120? in 97?
10. How many times is 11 contained in 33? in 77? in 121? in 88? in 100?
11. How many times is 12 contained in 60? in 132? in 36? in 96? in 117?
12. How many times is 7 contained in 17? in 3? in 38? in 62? in 29?
13. How many times is 8 contained in 53? in 7? in 71? in 90? in 21?
14. How many times is 9 contained in 23? in 100? in 48? in 80? in 10?
15. How many times is 12 contained in 16? in 37? in 140? in 101? in 92?
16. Florence has 47 questions in division to work in the week; how many must she do each day?
17. George has 56 apples and he wants to make them last 7 weeks; how many may he eat each week? how many each day?
18. Charlie wants to read a book, containing 135 pages, through in 11 days; how many pages must he read each day?
19. Emma has 78 books, and wishes to divide them as nearly as possible equally among 7 shelves? how many must she put on each shelf?
20. A farmer has 107 sheep, and wishes to divide equally, or as nearly so as possible, among 9 fields; how many must be placed in each?

RULE FOR SHORT DIVISION.

50. *Set down the divisor to the left of the dividend.*

Begin with the left-hand figure of the dividend and divide each figure in succession by the divisor, setting the quotient-figure directly under the figure divided. If there is a remainder after dividing any figure, prefix it to the next figure of the dividend, and divide the number thus formed as before. When the divisor is not contained once in any figure of the dividend, write a cipher under that figure and consider that figure as a remainder.

EXAMPLE 1.—Divide 271406 by 5.

OPERATION.
5)271406
54281⅕

Here we say 5 in 2, no times; but since it is the left-hand figure, we do not set down the 0 in the quotient; next 2 written before the 7 makes 27, 5's in 27, 5 and 2 over; 2 before 1 makes 21, 5's in 21, 4 and 1 over; 1 before 4 makes 14, 5's in 14, 2 and 4 over, &c. At the end we have a remainder of 1, and since we cannot actually divide 1 by 5, we indicate the division by writing it thus ⅕. (See Art. 48, Note.)

EXAMPLE 2.—Divide 704653 by 8.

OPERATION.
8)704653
88081⅝

Here we say 8's in 70, 8 and 6 over; 8's in 64, 8; 8's in 6, 0 and 6 over; 8's in 65, 8 and 1 over; 8's in 13, 1 and 5 over, and we set down this last remainder by writing it above a line with the 8 below it and read it five eighths.

EXERCISE 12.

(1)	(2)	(3)	(4)
2)7146987	3)8914567	4)914007	5)130047198

(5)	(6)	(7)	(8)
6)987650432	7)109147165	3)91807146	9)91471009

(9)	(10)	(11)	(12)
10)222333444	11)3121315161	12)914556677	9)111111111111

13. 9146291 ÷ 2
14. 714632 ÷ 3
15. 1234610 ÷ 4
16. 7000000 ÷ 5
17. 8100406 ÷ 6
18. 9001629 ÷ 7
19. 100610067 ÷ 8
20. 99999999 ÷ 10
21. 88888888 ÷ 12
22. 123456789 ÷ 9
23. 918276543 ÷ 11
24. 200000000 ÷ 7

25. *Divide* two hundred millions seven hundred and twenty-six thousand seven hundred and eleven *by* twelve.

26. If 7 horses cost $882, what is the cost of one horse?

27. If a field of 11 acres produces 746 bushels of oats, what is the yield per acre?

28. If 12 schools together contain 1932 scholars, how many is that on an average to each school?

51. To divide by a composite number, none of whose factors is greater than 12:—

RULE.

Divide the given dividend by one factor of the given divisor, and then divide the quotient thus obtained by the other factor.

To obtain the correct remainder, multiply the last remainder by the first divisor, and to the product add the first remainder.

EXAMPLE.—Divide 71469 by 35.

OPERATION.

5)71469
7)14293..4 = 1st rem.
2041..6 = 2d rem.
6 × 5 = 30 + 4 = 34
Ans. $2041\frac{34}{35}$.

Here the factors of the divisor are 5 and 7. In dividing by 5 we get a remainder, 4, and in dividing the first quotient by 7 we get a remainder, 6. Then, to get the true remainder, we multiply 6, the last remainder, by 5, the first divisor, and add 4, the first remainder, to the product. This gives us 34, which we write above the divisor, as before explained.

EXERCISE 13.

1. Divide 714967 by 16.
2. Divide 100901 by 27.
3. Divide 9186713 by 81.
4. Divide 16151712 by 144.
5. Divide 1671932 by 42.
6. Divide 2222222 by 108.
7. Divide 617149324 by 121.
8. Divide 8182838485 by 100.
9. Divide 66778991 by 64.
10. Divide 78998778998 by 54.
11. *Divide* nine hundred and seventeen millions forty-eight thousand and six *by* one hundred and ten.
12. *Divide* seventy millions four thousand and nineteen *by* sixty-three.
13. *How many times is* fifty-six *contained in* seventy-nine *times* four hundred and eleven thousand six hundred and nine?
14. A bushel of wheat weighs 60 lbs.; how many bushels are there in 71496 lbs.?
15. How many bushels of rye are there in 918674 lbs., one bushel of rye weighing 56 lbs.?
16. How many bushels of barley are there in 291717 lbs., one bushel of barley weighing 48 lbs.?
17. If 48 cows cost $1774, what is the cost of one cow?
18. If 21 bushels of pease weigh 1260 lbs., what is the weight o bushel?
19. Divide 71496 × 7 × 17 by 66.
20. Divide $71498 equally among 45 persons.

RULE FOR LONG DIVISION.

52. *Set down the divisor to the left of the dividend, as in short division, and the quotient to the right, thus:*

Divisor) Dividend (Quotient.

Find how many times the divisor is contained in the fewest figures of the dividend that will contain it once or more, and place the figure thus found in the quotient.

Multiply the divisor by the figure put in the quotient, write the product under the figures divided, and subtract.

To the right of the remainder thus obtained bring down the next figure of the dividend; divide the number thus formed as before, and proceed thus till all the figures of the dividend have been brought down.

When there is a remainder at the end of the process, write it over the divisor and annex it to the quotient.

PROOF OF DIVISION.—*Multiply the quotient by the divisor, and add in the remainder. The sum should be equal to the dividend.*

Example 1.—Divide 714986 by 613.

OPERATION.

$613)714986(1166\frac{228}{613}$
613
1019
613
4068
3678
3906
3678
228

Here the fewest figures that will contain 613, the divisor, are three, viz., 714. 613 in 714 will go 1 time, we therefore set 1 in the quotient; then *once* 613 is 613, which we set down under the 714 and subtract. We thus get a remainder of 101, to which we bring down the 9, and thus obtain 1019 as the new number to be divided. Next, 613 in 1019 will go one time; we therefore set down the *1* in the quotient, multiply and subtract as before, and thus obtain 406 for remainder, to which we bring down 8, the next figure of the dividend. This gives us 4068 for the next number to be divided; 613 into 4068 will go 6 times, etc.

Example 2.—Divide 896714 by 8842.

OPERATION.

$8842)896714(101\frac{3672}{8842}$
8842
12514
8842
3672

Here we say 8842 into 8967 will go once, and we thus get a remainder 125, to which we bring down the 1. This gives us 1251 for the next number to be divided; 8842 into 1251 will go 0 times, and we accordingly put 0 into the quotient and bring down the next figure, viz., 4, to the right of the 1251, and thus get 12514 as the number to be divided, etc.

Exercise 14.

1. 8916749÷227
2. 8161413÷1116
3. 1498706÷2106
4. 8222800÷8161
5. 7142347÷23
6. 6171112÷17
7. 8891876÷28161
8. 11223344÷3344
9. 91929394÷81007
10. 19167123÷19123

11. Divide 9167492 by 7 × 17 × 93.
12. Divide 6149811 by 13 × 15 × 11.
13. Divide 8182700 by 617 times 23.
14. The product is 2526426017908695, the multiplier is 27498766; what is the multiplicand?
15. The product of two numbers is 405768300; one of the numbers is 50406, what is the other?
16. What number multiplied by 538362 will make 4984155396?
17. 723 times 417 is how many times 917?
18. 238 times 1476 is how many times 91?
19. 271 times 777 is how many times 1027?
20. 1271 times 2986 is how many times 407 × 11?
21. If a ship sail 217 miles a day, how long will it require to complete a journey of 9142 miles?
22. If one acre of land cost 43 shillings, how much can be purchased for £1798?
23. *Divide* twenty-seven millions four thousand and nine *by* four thousand and seventeen.
24. *Divide* eight billions seventy millions and six *by* forty thousand six hundred and ninety.
25. *Divide* seven hundred and four millions and one *by* seventy-nine thousand four hundred and ten.
26. If 29 tons of hay cost $677, what will 1 ton cost?

(27)	(28)
11)123456789123456789123456789	9)914768134298764714155986777
(29)	(30)
12)778188909664422113332700614	7)120034005600780091400671478

NOTE.—The pupil should be exercised in these long lines by simply naming the divisor, partial dividend, and quotient-figure, as follows: 11..12..1, 11..13..1, 11..24..2, 11..25..2, 11..36..3, &c.

RECAPITULATION AND EXAMINATION QUESTIONS.

1. *Question.* What is Division?
   Answer. Division is the process of finding how often one number is contained in another.
2. *Q.* What is the dividend?
   A. The dividend is the number to be divided.
3. *Q.* What is the divisor?
   A. The divisor is the number by which we divide.
4. *Q.* What is the quotient?
   A. The quotient is the result of the division, and expresses how many times the divisor is contained in the dividend.

5. *Q.* What is the remainder?
A. The remainder is what is left when the divisor is not contained an exact number of times in the dividend.

6. *Q.* How is the remainder written?
A. We write the remainder above a short horizontal line with the divisor beneath it, and annex the expression thus formed to the integral part of the quotient.

7. *Q.* Can the remainder be as great as the divisor?
A. The remainder cannot be as great as the divisor.

8. *Q.* How many modes are there of expressing the division of one number by another?
A. We have three modes of expressing the division of one number by another, viz.: by writing between the two numbers the sign of division, ÷ or either of its parts : or —. Thus, if we wish to express the division of 1798 by 16, we may do it thus, 1798÷16, or thus 1798 : 16, or thus $\frac{1798}{16}$.

9 *Q.* What is the distinction between short division and long division?
A. It is short division when the divisor is not greater than 12 and long division when the divisor is greater than 12.

10. Give the rule for short division. (See Art. 50.)

11. Give the rule for division when the divisor can be broken up into two factors, neither of which is greater than 12. (See Art. 51.)

12. *Q.* In this last rule, when there is a remainder after either division, how is the correct remainder found?
A. To find the true remainder we multiply the first divisor by the last remainder and add in the first remainder.

13. Give the rule for long division. (See Art. 52.)

14. *Q* In long division, how can you tell how many times the divisor is contained in the part of the dividend under consideration?
A. By asking how many times the first figure of the divisor will go into the first figure, or first two figures, of the dividend.

15. *Q.* How can you tell when the figure put in the quotient is too large or too small?
A. If it be too large, the product of the divisor by it will be greater than the part of the dividend used; if too small, the remainder will be greater than the divisor.

16. *Q.* How do we prove division?
A. To prove division we multiply the divisor and the quotient together and to the product add the remainder, if there be any. The result should be the dividend.

17 *Q.* How do we divide by 10, 100, 1000, &c.?
A. We divide by 10 by cutting off the right-hand figure of the dividend: by 100, by cutting off the last two digits to the right; by 1000, by cutting off the last three digits, &c.

SECTION II.

DECIMAL CURRENCY, TABLES OF MONEY, WEIGHTS, AND MEASURES, REDUCTION, AND COMPOUND RULES.

DECIMAL CURRENCY.

1. The denominations of Canadian money are dollars and cents, and 100 cents make 1 dollar. The following explains the mode of writing and reading sums of money expressed in the decimal currency:

$7·00 is read 7 dollars.
$9·20 " 9 dollars and 20 cents.
$16·89 " 16 dollars and 89 cents.
$417·23 " 417 dollars and 23 cents.
423 dollars and 17 cents is written $423·17
94 dollars and 99 cents " $94·99
6149 dollars and 67 cents " $6149·67

2. Dollars are converted into cents by annexing two ciphers.

Thus, $69 = 6900 cents. $479 = 47900 cents.
$17 = 1700 cents. $2161 = 216100 cents.

3. Cents are converted into dollars by cutting off the two right-hand figures. These figures are cut off by placing a small dot between the second and third figures from the right-hand side. When thus cut off, the figures to the left of the dot are dollars, those to the right of the dot, cents. Thus:—

71934 cents = $719·34
4290 cents = $42·90
291671 cents = $2916·71

EXERCISE.

1 Read $17·42 ; $716·00 ; $918·19 ; $606·06.
2 Read $79·40 , $916·87½ ; $4·64 ; $70·09.
3. Read $712·20 , $98·89½ ; $11·10 , $617·08½

4. Write down in figures ninety-three dollars forty-seven cents.
5. Write down six hundred and nine dollars and twelve cents.
6. Write down four hundred and thirty dollars and eight cents.
7. Write down seven thousand and seventy dollars and seventeen cents.
8. How many cents are there in three dollars?
9. How many cents are there in seventeen dollars and ninety-eight cents?
10. How many cents are there in $6194·17?
11. Reduce $471·29 to cents.
12. Reduce $17·43 to cents.
13. How many dollars are there in 16714 cents?
14. How many dollars are there in 9009 cents?
15. How many dollars are there in 6714927 cents?
16. Reduce 17147 cents to dollars and cents.
17. Reduce 6147 cents to dollars and cents.
18. Reduce 98765 cents to dollars and cents.

---

4. To reduce old Canadian money (pounds, shillings, and pence) to the new or decimal currency:—

RULE.

Multiply the pounds by 400, *the shillings by* 20, *and the farthings in the given pence and farthings by* $\frac{5}{12}$.

Add the three products together, and the sum will be the answer in cents.

NOTE.—We multiply by $\frac{5}{12}$ by multiplying by 5 and dividing the product by 12.

EXAMPLE 1.—Reduce £79 4s. 11¾d. to dollars and cents.

OPERATION.

$79 \times 400 = 31600$ = cents in £79.
$4 \times 20 = 80$ = cents in 4s.
$47 \times 5 \div 12 = 19\frac{7}{12}$ = cents in 11¾d.
Sum = $\$316{\cdot}99\frac{7}{12}$ = dollars and cents in £79 4s. 11¾d.

REASON.—£1 = $4 = 400 cents; 1s. = 20 cents; and 12 farthings = 5 cents, or one farthing = $\frac{5}{12}$ of a cent.

EXAMPLE 2.—Reduce £217 11s. 9½d. to dollars and cents.

OPERATION.

$217 \times 400 = 86800$ = cents in £217.
$11 \times 20 = 220$ = cents in 11s.
$38 \times 5 \div 12 = 15\frac{5}{6}$ = cents in $9\frac{1}{2}$d.
Sum = $\$870{\cdot}35\frac{5}{6}$ = dollars and cents in £217 11s. $9\frac{1}{2}$d.

EXERCISE 15.

1. Reduce £719 10s. $4\frac{3}{4}$d. to dollars and cents.
2. Reduce £671 12s. 8d. to dollars and cents.
3. Reduce £167 0s. $10\frac{1}{4}$d. to dollars and cents.
4. Reduce £17 17s. $7\frac{1}{2}$d. to dollars and cents.
5. Reduce £655 19s. $8\frac{3}{4}$d. to dollars and cents.
6. Reduce £777 11s. 3d. to dollars and cents.
7. Reduce £111 11s. 11d. to dollars and cents.
8. Reduce £567 8s. $9\frac{1}{4}$d. to dollars and cents.
9. How many dollars and cents are there in £57 8s. $5\frac{1}{2}$d.?
10. How many dollars and cents are there in £704 19s. $11\frac{3}{4}$d.?

---

5. To reduce dollars and cents to pounds, shillings, and pence, old Canadian currency:—

RULE.

Divide the dollars by 4, *and call the quotient pounds. Reduce the dollars in the remainder to cents, and to them add the given cents; then divide the number of cents thus obtained by* 20, *and call the result shillings. Lastly, multiply the remaining cents by* 3, *and divide the product by* 5; *the quotient is pence.*

EXAMPLE 1.—Reduce \$279·10 to pounds, shillings, and pence.

OPERATION.

$\$279 \div 4 = £69$ and a remainder of \$3.
$\$3{\cdot}10 = 310$ cents, and $310 \div 20 = 15$ shillings and a remainder of 10 cents.
$10 \times 3 = 30 \div 5 = 6$d.
Hence \$279·10 = £69 15s. 6d.

REASON.—\$4 = £1; 20 cents = 1 shilling; and 5 cents = 3 pence.

EXAMPLE 2.—Reduce $\$71{\cdot}29\frac{7}{12}$ to pounds, shillings, and pence.

OPERATION.

$\$71 \div 4 = £17$ and a remainder of \$3.
$\$3{\cdot}29\frac{7}{12} = 329\frac{7}{12}$ cents $\div 20 = 16$ shillings and a remainder of $9\frac{7}{12}$ cents.
$9\frac{7}{12} \times 3 = 28\frac{3}{4} \div 5 = 5\frac{3}{4}$d.
Hence $\$71{\cdot}29\frac{7}{12} = £17\ 16$s. $5\frac{3}{4}$d.

EXERCISE 16.

1. Reduce \$719·40 ; \$917·10 ; \$69·70, and \$417·95, to pounds, shillings, and pence.
2. Reduce \$171·11 ; \$190·09 ; \$1674·23, and \$777·77, to pounds, shillings, and pence.
3. Reduce \$444·44 ; \$111·26 ; \$70·07, and \$191·82, to pounds, shillings, and pence.
4. How many pounds, shillings, and pence are there in \$714·23? in \$21·17? in \$16·16? in \$7934·08?

6. Since the denominations of decimal money increase according to the scale of ten, the foregoing rules are applicable, without any alteration, to the addition, subtraction, multiplication, and division of dollars and cents.

EXAMPLE 1.—Add together \$719·42, \$917·87, \$429·84, and \$918·76.

OPERATION.	
\$719·42 917·87 429·84 918·76 ——— \$2985·89	Here the sum of the first or right-hand column is 19, and we put down the 9 and carry the 1; the sum of the second column with the 1 carried is 28, we set down the 8 and carry the 2; the sum of the third column with the 2 carried is 35, we set down the 5 and carry the 3, &c.

EXAMPLE 2.—From \$9147·86 take \$871·94.

OPERATION.	
\$9147·86 871·94 ——— \$8275·92	Here we say 4 from 6 and 2 remain; 9 from 8 we cannot, borrow 1 from the 7 in the third column; then 9 from 18 and 9 remain; 1 from 6 (or 2 from 7) and 5 remain, &c.

EXAMPLE 3.—Multiply $67·42 by 247.

OPERATION.

```
 $67·42
    247
 ------
  47194
 26968
13484
-------
$16652·74
```

Here we consider the $67·42 as being 6742 cents, i. e., we pay no attention to the separating point in the multiplicand, and merely point off the two right-hand figures in the product, for cents.

EXAMPLE 4.—Divide $7149·80 by 19.

OPERATION.

$19)7149·80(376·30\frac{10}{19}$

```
19)7149·80(376·30
   57
   ---
   144
   133
   ---
    119
    114
    ---
      5.8
      5.7
      ---
        10
```

Here we divide without regarding the separating point, except that, when we bring down the first figure to the right of the point in the dividend, we place a point in the quotient.

EXAMPLE 5.—Divide $7194·76 by $29·34.

OPERATION.

$29·34)7194·76(245\frac{646}{2934}$

```
29·34)7194·76(245
      5868
      ------
      1326·7
      1178·6
      -------
       153·16
       146·70
       ------
         6·46
```

Here we divide without regarding the separating point, i. e., we consider the question as being how often is 2934 cents contained in 719476 cents. We get as a result $245\frac{646}{2934}$ times, or 245 times with a remainder of $6·46.

EXERCISE 17.

1. What is the sum of $740·83, $614·91, $9167·14, $918·40, $21·74, $614·20, and $29·78?
2. What is the sum of $888·77, $916·66, $1147·98, $91867·42, $1919·19, $981·92, and $444·59?

3. Add together $617·49, $74·27, $23·32, and £9 8s. 7½d.

4. From $6714·98 take $982·49.

5. From $4216·23 take $2437·86.

6. What is the difference between $914·71 and £471 16s. 10½d.?

7. What is the difference between £29 18s. 9d. and $649·32?

8. Multiply $671·21 by 48.

9. Multiply $519·26 by 789.

10. How much is 529 times $16·83?

11. Divide $6149·73 by 67.

12. Divide $18793·67 by 149.

13. Divide $1714·86 by $71·42.

14. Divide $9167·42 by $147·83.

15. Purchased a horse for $147·80, a carriage for $217·20, harness for $63·27, and a saddle for $23·87; what did the whole cost me?

16. What is the twenty-seventh part of $916·74?

17. Divide $671·90 equally among 13 persons; what is the share of each?

18. I went out to make purchases, having with me £71 16s. 7½d. I bought and paid for groceries, $17·80; dry goods, $21·63; furniture, $123·76; and books, $37·26. How much change did I bring home?

19. What is the cost of 17 tons of hay at $17·45 per ton?

20. From $723·86 take $297·42; multiply the result by 63 and divide the product by 217.

21. In 1858 the exports of Canada were as follows: Produce of the mine, $314823; produce of the fisheries, $718296; produce of the forest, $9447727; animals and their products, $2462765; agricultural products, $7904400; manufactures, $325376; other articles, $112538; and goods not reported (estimated) $1443044. What was the total value of the Canadian exports for 1858?

22. The imports into Canada for the year 1858 amounted to $29078527; how much did the Canadian imports exceed the exports in 1858?

23. If the population of Canada be 2954600, what was the value of the imports for each individual in 1858?

EXAMINATION QUESTIONS.

NOTE.—The answers to these questions are found as indicated after each question.

1. What are the denominations of Canadian money? (Art. 1.)

2. What are the Canadian silver coins? (Table on next page.)

3. What is the diameter of the Canadian cent? (Table on next page.)

4. How many cents are equal in weight to 1 lb. Avoirdupois? (Table below.)

5. Why is Canadian and United States money called *decimal?* (*Ans.* From the Latin word *decem*, "ten," because the orders increase in a tenfold ratio; i. e., 10 mills make 1 cent, 10 cents 1 dime, 10 dimes 1 dollar, &c.)

6. How are dollars converted into cents? (Art. 2.)

7. How are cents converted into dollars? (Art. 3.)

8. Give the rule for reducing old Canadian currency into dollars and cents. (Art. 4.) Give the reason for the process.

9. Give the rule for reducing dollars and cents to old Canadian currency. (Art. 5.) Give the reason for the process.

10. How do we add and subtract, multiply and divide dollars and cents?

    (*Ans.* We regard the dollars and cents as so many cents, and then proceed as in the simple rules.)

TABLES OF MONEY, WEIGHTS AND MEASURES.

CANADIAN DECIMAL MONEY.

100 cents (c.) make 1 dollar, marked $.

NOTE.—The coins are a five-cent piece, a ten-cent piece, and a twenty-cent piece, all of which are silver; and a one-cent piece, which is bronze.

The one-cent piece is exactly one inch in diameter, and 100 cents weigh 1 lb. Avoirdupois.

OLD CANADIAN CURRENCY.

TABLE.

4 farthings	make	1 penny,	marked	d.
12 pence	"	1 shilling,	"	s.
5 shillings	"	1 dollar,	"	$.
4 dollars	"	1 pound,	"	£.

ENGLISH OR STERLING MONEY.

TABLE.

4 farthings (qr.)	make	1 penny,	marked	d.
12 pence	"	1 shilling,	"	s.
20 shillings	"	1 pound,	"	£.

NOTE.—The Guinea is equal to 21 shillings and the Sovereign to 20 shillings sterling.

UNITED STATES MONEY.

TABLE.

10 mills (m.)	make	1 cent,	marked	ct.
10 cents	"	1 dime,	"	d.
10 dimes	"	1 dollar,	"	$.
10 dollars	"	1 eagle,	"	E.

AVOIRDUPOIS WEIGHT.

TABLE.

16 drams	make	1 ounce,	marked	oz.
16 ounces	"	1 pound,	"	lb.
25 pounds	"	1 quarter,	"	qr.
4 quarters	"	1 hundredweight,	"	cwt.
20 cwt.	"	1 ton,	"	t.

Note.—This weight is used in weighing heavy articles, as meat, groceries, vegetables, grain, etc.

TROY WEIGHT.

TABLE.

24 grains (grs.)	make	1 pennyweight,	marked	dwt.
20 pennyweights	"	1 ounce,	"	oz.
12 ounces	"	1 pound,	"	lb.

Note.—Troy weight is used in weighing the precious metals and stones; also in scientific investigations.

APOTHECARIES' WEIGHT.

TABLE.

20 grains (grs.)	make	1 scruple,	marked	scr. or ℈.
3 scruples	"	1 dram,	"	dr. or ʒ.
8 drams	"	1 ounce,	"	oz. or ℥.
12 ounces	"	1 pound,	"	lb.

Note.—Apothecaries and Physicians mix their medicines by this weight, but they buy and sell by Avoirdupois.

LONG MEASURE.

TABLE.

12 lines (l.)	make	1 inch,	marked	in.
12 inches	"	1 foot,	"	ft.
3 feet	"	1 yard,	"	yd.
5½ yards	"	1 rod, pole, or perch,	"	rd. or per
40 rods or perches	"	1 furlong,	"	fur.
8 furlongs	"	1 mile,	"	m.
3 miles	"	1 league,	"	lea.

NOTE.—The degree, or 360th part of the circumference of the earth, is about $69\frac{1}{6}$ miles.

4 inches	make	1 hand (used in measuring horses).
18 inches	"	1 cubit.
3 feet	"	1 pace.
6 feet	"	1 fathom.
120 fathoms	"	1 cable-length.

SQUARE OR LAND MEASURE.

TABLE.

144 square inches	make	1 square foot,	marked	sq. ft.
9 square feet	"	1 square yard,	"	sq. yd.
30¼ square yards	"	1 square rod,	"	sq. rd.
40 square rods	"	1 rood,	"	r.
4 roods	"	1 acre,	"	a.
640 acres	"	1 square mile,	"	sq. m.

NOTE.—Square Measure is used in measuring surfaces, as, for example, in estimating the work of painters, plasterers, pavers, etc.; also in measuring land.

In measuring land, Gunter's chain is used. It is divided into 100 links.

$7\frac{92}{100}$ inches	make	1 link,	marked	l.
100 links or 4 rods	"	1 chain,	"	c.
80 chains	"	1 mile,	"	m.
10000 square links	"	1 sq. chain,	"	sq. c.
10 square chains	"	1 acre,	"	a.

CUBIC OR SOLID MEASURE.

TABLE.

1728 cubic inches (cub. in.) make 1 cubic foot, marked cub. ft.

27 cubic feet make 1 cubic yard, marked cub. yd.

40 cubic feet of round timber } make 1 ton, marked
50 cubic feet of hewn timber } ton.

128 cubic feet of firewood make 1 cord, marked c.

A pile of cord-wood 4 feet high, 4 feet wide, and 8 feet long, contains 128 cubic feet, or 1 cord. One foot in length of such a pile is called a *cord-foot;* it is equal to 16 solid feet, and is consequently equivalent to the eighth part of a cord.

CLOTH MEASURE.

TABLE.

2¼ inches (in.)	make	1 nail,	marked	na.
4 nails	"	1 quarter,	"	qr.
3 quarters	"	1 Flemish ell,	"	Fl. e.
4 quarters	"	1 yard,	"	yd.
5 quarters	"	1 English ell,	"	E. e.
6 quarters	"	1 French ell,	"	F. e.

NOTE.—The Scotch ell contains 4 quarters 1⅕ inch.

DRY MEASURE.

TABLE.

2 pints (pt.)	make	1 quart,	marked	qt.
4 quarts	"	1 gallon,	"	gal.
2 gallons	"	1 peck,	"	pk.
4 pecks	"	1 bushel,	"	bu.
36 bushels	"	1 chaldron,	"	ch.

NOTE.—This measure is used in buying and selling vegetables, fruits, grains, &c.

LIQUID MEASURE.

TABLE.

4 gills (gill)	make	1 pint,	marked	pt.
2 pints	"	1 quart,	"	qt.
4 quarts	"	1 gallon,	"	gal.
31½ gallons	"	1 barrel,	"	bar.
2 barrels	"	1 hogshead,	"	hhd.
2 hogsheads	"	1 pipe,	"	pi.
2 pipes	"	1 tun,	"	tun.

TIME MEASURE.

TABLE.

60 seconds (sec.)	make	1 minute,	marked	min.
60 minutes	"	1 hour,	"	h.
24 hours	"	1 day,	"	d.
7 days	"	1 week,	"	wk.
4 weeks	"	1 lunar month,	"	mo.
13 lunar months or 12 calendar months or 365½ days (nearly)	make	1 civil year,	marked	yr.

The names of the months and the number of days in each are as follows:

First	month,	January,	has	31	days.
Second	"	February,	"	28	"
Third	"	March,	"	31	"
Fourth	"	April,	"	30	"
Fifth	"	May,	"	31	"
Sixth	"	June,	"	30	"
Seventh	"	July,	"	31	"
Eighth	"	August,	"	31	"
Ninth	"	September,	"	30	"
Tenth	"	October,	"	31	"
Eleventh	"	November,	"	30	"
Twelfth	"	December,	"	31	"

The number of days in the respective months may be recalled by recollecting the following well-known lines:

Thirty days has September,
April, June, and November;
February has twenty-eight alone—
All the rest have thirty-one;
But leap-year coming once in four,
February then has one day more.

The number of days in each month may also be recollected by counting the months on the *four* fingers and the *three* intervening spaces. Thus, January on first finger, February on space between first and second fingers, March on second finger, April in second space, May on third finger, June in third space, July on fourth finger, August on first finger, (since there are no more spaces,) September in first space, &c. Now, when counted thus, all the months having 31 days come on the fingers, and February and all having 30 only, fall into the spaces.

CIRCULAR MEASURE.

TABLE.

60 seconds (″)	make	1 minute,	marked ′.
60 minutes	“	1 degree,	“ °.
30 degrees	“	1 sign,	“ s.
12 signs or 360 degrees	“	the circumference of a circle,	marked c.

NOTE.—This, which is sometimes called *Angular Measure*, is used by astronomers, navigators, and surveyors for measuring angles, and for reckoning latitude and longitude.

MISCELLANEOUS TABLE.

12 individual things	make	1 dozen.
12 dozen	“	1 gross.
12 gross..............	“	1 great gross.
20 individual things	“	1 score.
24 sheets of paper .	“	1 quire.
20 quires............	“	1 ream.
200 pounds...........	“	1 barrel of pork or beef.
196 “	“	1 barrel of flour.
14 “	“	1 stone.

BOOKS.

A sheet folded into two leaves is called a *folio*.
“ folded into four leaves is called a *quarto*, or 4to.
“ folded into eight leaves is called an *octavo*, or 8vo.
“ folded into twelve leaves is called a *duodecimo*, or 12mo.
“ folded into sixteen leaves is called a 16mo.
“ folded into eighteen leaves is called an 18mo.

REDUCTION.

7. Reduction is the process of changing a number from one denomination to another without altering its value.

8. Reduction Ascending is the process of reducing a number from a lower to a higher denomination.

9. Reduction Descending is the process of reducing a number from a higher to a lower denomination.

RULE FOR REDUCTION DESCENDING.

10. *Multiply the highest given denomination by that quantity which expresses the number of the next lower contained in one of its units, and add to the product that number of the next lower denomination which is found in the quantity to be reduced.*

Proceed in the same way with the result, and continue the process until the required denomination is obtained.

Example 1.—Reduce 427 miles to yards.

OPERATION.

427 = miles
8
3416 = furlongs
40
136640 = perches
5½
683200
68320
751520 = yards.

Here we first multiply by 8, because each mile is equal to 8 furlongs; next we multiply the furlongs by 40, to reduce them to perches, because each furlong is equal to 40 perches; lastly we multiply the perches by 5½, to reduce them to yards.

Example 2.—Reduce 6 bushels 3 pks. 1 gal. 1 qt. to quarts.

OPERATION.

6 bush. 3 pks. 1 gal. 1 qt.
4
27 = pks. in 6 bush. 3 pks.
2
55 = gals. in 6 bush. 3 pks. 1 gal.
4
221 = qts. in 6 bush. 3 pks. 1 gal. 1 qt.

Here we first multiply the 6 bushels by 4 to reduce to pecks and add in the 3 pecks given; next we multiply the resulting pecks by 2 to reduce them to gallons, and add in the 1 gallon given, &c.

Exercise 18.

1. Reduce 47 cords of wood to cubic feet.
2. Reduce 6497 lbs. Avoir. to ounces.
3. Reduce £97 16s. 8¾d. to farthings.
4. Reduce 127 a. 2 r. 17 per. 19 yds. 8 ft. 121 in. to inches.
5. Reduce 569 tons 4 cwt. 3 qrs. 17 lbs. 4 oz. 7 drs. to drams.
6. Reduce 4 pipes 1 hhd. 1 brl. 19 gals. 2 qts. to quarts.
7. Reduce 17 miles 7 fur. 7 per. 2 yds. 2 ft. 4 in. to lines
8. Reduce 5° 17′ 49″ to seconds.
9. Reduce 2 ch. 17 bush. 2 pks. 1 gal. 1 qt. to pints.
10. Reduce 9 French ells 1 qr. 3 na. 1¼ in. to inches.
11. Reduce 17 weeks 4 days 9 hours 29 min. 17 sec. to seconds.
12. Reduce 29 E. 9 dollars 6 dimes 2 cents 4 mills to mills.
13. Reduce 17 lbs. 9 oz. 16 dwt. 11 grs. to grains.
14. Reduce 37 cub. yds. 9 cub. ft. 1111 cub. in. to inches.
15. Reduce 129 lbs. 4 oz. 2 scr. 11 grs. to grains.
16. How many square feet are there in 127 square perches?
17. How many inches are there in 127 Eng. ells 1 qr. 2 na.?
18. How many cub. ft. of wood are there in 17 cords 63 cub. ft.?
19. How many quarts are there in 714 gallons?
20. How many scruples are there in 71 lbs. 11 oz. 3 drs.?
21. In 16 cwt. 1 qr. 19 lbs. how many ounces are there?
22. In 11 miles 2 ft. how many inches are there?
23. In 123 acres 17 per. how many square yards are there?
24. In 27 years 16 days 4 min. how many seconds are there?
25. In 161 days 14 hours how many hours are there?
26. In £1978 17s. 9¾d. how many farthings are there?
27. How many pints are there in 17 bush. 1 pk. 1 gal.?
28. How many grains are there in 9 lbs. 17 dwt.?
29. Reduce 9 sq. m. 1 a. 0 r. 9 yds. to square inches.
30. Reduce £171 11s. 1¼d. to farthings.

RULE FOR REDUCTION ASCENDING.

11. *Divide the given number by that number which it takes of the given denomination to make one of the next*

higher. Set down the remainder, if any, and proceed in the same manner with each successive denomination till you come to the one required. The last quotient, with the several remainders annexed, will be the answer required.

EXAMPLE 1.—How many pounds Apoth. are there in 16719 scruples?

OPERATION.

3)16719 scr.
8)5573 drs.
12)696 oz. 5 drs.
58 lbs. 0 oz. 5 drs.

Here we divide the scr. by 3 to reduce to drams, because every 3 scruples make 1 dram. We thus obtain 16719 scr. = 5573 drams. Next we divide the drams by 8, because every 8 drams are equal to 1 oz., and we thus find the given number of scruples to be equal to 696 oz. 5 drs. 0 scr. Finally, we divide the ounces by 12, and thus obtain 58 lbs. 0 oz. 5 drs. for the answer.

EXAMPLE 2.—Reduce 719864 pints to bushels.

OPERATION.

2)719864 = pints.
4)359932 qts. 0 pts.
2)89983 gals. 0 qts. 0 pts.
4)44991 pks. 1 gal. 0 qts. 0 pts.
11247 bush. 3 pks. 1 gal. 0 qts. 0 pts. *Ans.*

EXERCISE 19.

1. Reduce 71980 inches to miles, furlongs, &c.
2. Reduce 6142 minutes to weeks, days, &c.
3. Reduce 81427 grains to pounds, ounces, &c., Apoth. weight.
4. Reduce 9141762 cubic inches of wood to cord-feet, &c.
5. In 7177 pints how many chaldrons, bushels, &c.?
6. In 914 cubic feet how many cubic yards?
7. In 61479 inches how many French ells, qrs. &c.?
8. In 89 days how many weeks, &c.?
9. How many tons, cwts., &c., are there in 1714964 drams?
10. How many acres, roods, &c., are there in 1714961 inches?

11. How many tuns are there in 171439 quarts?
12. How many years, (each 365¼ days,) days, &c., are there in 171491642 seconds?
13. Reduce 171496894 farthings to pounds, shillings, and pence.
14. Reduce 2987149 mills to eagles, dollars, dimes, &c.
15. Reduce 21111496 inches to roods, square perches, &c.
16. Reduce 17498 cubic feet of wood to cords.
17. Reduce 919817 pence to pounds, shillings, &c.
18. Reduce 999 dwt. to pounds, &c.
19. Reduce 1771 gallons to bushels.
20. Reduce 91666 Flemish ells to French ells.
21. How many cwts. qrs. and lbs. are there in 17149 lbs.?
22. How many miles, fur. per. &c., are there in 17110 feet?
23. How many degrees, min. and sec. &c., are there in 1111111 sec.?
24. Reduce 667789 cubic inches to cubic yards, &c.
25. Reduce 7891427 grains to pounds, Apoth.
26. Reduce 678846 grains to pounds, Troy.
27. Reduce 298714 drams to pounds, Avoir.
28. In 61479867 square inches how many acres, roods, &c.?
29. In 91999 yards how many leagues?
30. In 714986 inches how many fathoms

COMPOUND ADDITION.

12. Compound Addition is the addition of applicate numbers of more than one denomination.

RULE.

Set down the addends under one another so that units of the same order shall be in the same vertical column.

Begin at the right-hand side and add the first column; divide the sum by the number of that order which make one of the next higher; set down the remainder, if any, under the column added, and carry the quotient to the next column. Proceed thus through all the columns to the last.

EXAMPLE 1.—Add together 9 weeks 4 days 17 hours 11 min.; 6 wks. 3 days 11 hrs. 49 min.; 9 wks. 2 days 0 hrs. 53 min.; and 17 wks. 5 days 21 hrs. 35 min.

OPERATION.

wks.	ds.	hrs.	min.
9	4	17	11
6	3	11	49
9	2	0	53
17	5	21	35
43	2	3	28

Here the minutes added up amount to 148, which we divide by 60 in order to reduce them to hours; this gives 2 hours to carry to the next column and 28 minutes to set down in the column of minutes, and so on.

EXAMPLE 2.—What is the sum of £917 16s. 4¾d.; £216 11s. 11½d.; £160 14s. 7d.; £916 7s. 9¾d.; £100 0s. 9¼d.; £70 17s. 11½d.; and £16 16s. 9¼d.?

OPERATION.

£	s.	d.
917	16	4¾
216	11	11½
160	14	7
916	7	9¾
100	0	9¼
70	17	11½
16	16	9¼
2399	6	3

Here the farthings added amount to 12, which we divide by 4 to reduce them to pence; this gives us 3 pence to carry and no farthings to set down. The pence column, with the 3 carried, amounts to 63, which we divide by 12 to reduce to shillings; this gives us 3 pence to set down under the column added and 5 shillings to carry to the shillings' column, and so on.

EXERCISE 20.

(1)

£	s.	d.
179	11	4¼
96	2	0¾
297	8	11¾
9	0	10½
607	19	2½
96	17	8¾

(2)

miles	fur.	per	yds.
63	7	16	2
19	6	11	4
7	0	36	5
29	2	6	2½
11	6	23	4½
63	7	2	1

(3)

lbs.	oz.	drs.	scr.
16	11	4	1
9	8	5	0
126	7	4	2
91	8	7	1
6	0	2	0
27	4	6	2

(4)

cwt.	qrs.	lbs.	oz.
91	2	22	12
16	1	24	7
9	0	11	15
66	1	2	7
29	0	1	6

(5)

yds.	ft.	in
27	2	11
16	1	2
98	2	6
7	1	10
9	2	8

(6)

bush.	pks.	gals.
9	1	1
17	0	1
19	1	0
37	1	1
96	1	0

(7)

cords	cord ft.	cub. ft.
17	0	15
19	7	11
29	6	8
37	4	14

(8)

a.	r.	per.
297	3	16
96	0	9
11	2	39
27	3	16

(9)

yds.	qrs.	na.	in.
6	2	1	2
7	3	3	1
6	2	1	2
7	1	3	1½

(10)

lbs.	oz.	dwt.	grs.
16	4	2	17
93	11	17	23
38	10	16	14
27	0	9	12

(11)

gals.	qts.	pts.
12	1	1
16	0	1
10	1	1
9	1	0

(12)

sq. per.	sq. yds.	sq. ft.
17	23	6
98	16	7
81	30	6
27	27	2

(13)

wks.	days	hrs.
27	4	23
19	6	17
11	4	9
21	3	12
19	5	14

(14)

£	s.	d.
129	6	11¾
17	14	2½
93	11	7
16	19	2¼
9	2	9½

(15)

qrs.	lbs.	oz.	drs.
16	24	11	14
93	10	14	7
27	21	13	14
21	16	15	2
9	2	10	11

(16)

Eng. ells	qrs.	na.
27	1	1
43	2	2
91	3	2
16	1	3

(17)

pks.	gals.	qts.
12	1	0
6	1	1
12	1	1
19	1	1

(18)

oz.	dwt.	grs.
127	14	6
93	5	21
91	17	17
127	12	18

(19)

£	s.	d.
127	19	8
67	4	11¾
91	16	2¾
127	11	4¼
63	12	10½

(20)

r.	per.	yds.
127	16	20
19	17	30
63	27	16
47	35	9
63	10	25

(21)

yrs.	wks.	days
27	50	2
93	16	4
11	2	6
23	14	0
67	47	5

(22)

lbs.	oz.	drs.	scr.	grs.
16	11	7	2	17
96	2	3	1	19
126	10	4	0	16
46	11	5	2	16

(23)

a.	r.	per.	yds.	ft.	in.
27	2	36	7	7	27
43	3	37	8	6	126
123	2	4	6	1	107
86	0	13	27	6	23

(24)

lea.	miles	fur.	per.	yds.	ft.	in.	lines
14	2	7	23	4	1	10	7
16	1	3	16	2	0	6	10
7	1	6	33	5	2	6	4
17	2	6	17	3	2	7	11
16	2	7	15	2	2	8	9

COMPOUND SUBTRACTION.

13. Compound Subtraction is the subtraction of applicate numbers of more than one denomination.

RULE.

Set the subtrahend under the minuend so that units of the same order come in the same vertical column.

Begin at the right-hand side and subtract the first term of the lower line from the corresponding term of the upper line, if possible; but if not, increase the term of the upper line by the number of units of that denomination which make one of the next higher; then subtract and set the remainder under the first column and carry one to the given number of the next denomination contained in the subtrahend.

Proceed thus through all the columns to the last.

Example 1.—Subtract 27 miles 7 fur. 6 per. from 93 miles 2 fur. 7 per.

OPERATION.

miles	fur.	per.
93	2	7
27	7	6
65	3	1

Here we say 6 per. from 7 per. and 1 per. remains, and we set down this remainder under the column subtracted. Next, 7 fur. from 2 fur. we can't; and since 8 fur. make 1 mile, we increase the 2 fur. by 8; then 7 fur. from 10 fur. and 3 fur. remain. Again, adding 1 to the 7, we say 8 from 3 we can't; but 8 from 13, &c.

EXAMPLE 2.—From 27 lbs. 4 oz. 7 drs. 2 scr. take 9 lbs. 6 oz. 16 grs.

OPERATION.

lbs.	oz.	drs.	scr.	grs.
27	4	7	2	0
9	6	0	0	16
17	10	7	1	4

Here 16 grs. from 0, we can't; 16 from 20 and 4 remain; 1 from 2 and 1 remains; 0 from 7 and 7 remain; 6 from 4, we can't, but 6 from 16, i. e. 4 + 12 and 10 remains, &c.

EXERCISE 21.

(1)

days	hrs.	min.
147	20	3
49	16	4

(2)

miles	fur.	per.
167	6	3
93	7	27

(3)

qrs.	lbs.	oz.
13	13	6
11	22	11

(4)

gals.	qts.	pts.
729	0	0
247	1	1

(5)

a.	r.	per.
472	1	17
93	3	27

(6)

£	s.	d.
279	0	1¼
114	17	11½

(7)

lbs.	oz.	dwt.
176	3	4
97	5	17

(8)

yds.	qrs.	na
176	3	2
97	3	3

(9)

hrs.	min.	sec.
274	52	9
98	57	14

(10)

cords	cord ft.	cub. ft.
276	4	14
98	6	15

(11)

oz.	drs.	scr.
47	6	1
19	7	2

(12)

fur.	per.	yds.
16	23	4
11	23	5

(13)

cwt.	qrs.	lbs.
167	2	20
93	1	24

(14)

bush.	pks.	gal.
126	1	0
63	1	1

(15)

yrs.	wks.	days
163	42	6
93	47	2

(16)

r.	per.	yds
127	30	20
67	31	26

(17)

£	s.	d.
167	4	3
109	17	8¼

(18)

oz.	dwt.	grs.
274	6	2
193	7	17

(19)

£	s.	d.
2[illegible]1	14	6
136	14	11¼

(20)

cub. yds.	cub. ft.	cub. in.
146	7	1200
93	21	1467

(21)

hhds.	bar.	gal.
173	1	20
91	0	27

(22)

drs.	scr.	grs.
167	0	7
93	1	19

(23)

Flem. c.	qrs.	na.
16	0	0
9	2	3

(24)

sq. per.	sq. yds.	sq. ft.
167	14	3
119	27	7

COMPOUND MULTIPLICATION.

14. Compound Multiplication is the multiplication of applicate numbers of more than one denomination.

15. When the multiplicand does not exceed 12:—

RULE.

Set down the multiplier under the right-hand term.

Multiply every order of units in the multiplicand in succession, beginning with the lowest, by the multiplier, and divide each product, so formed, by the number of that denomination which makes one unit of the next higher: write down each remainder under units of its own order, and carry the quotient to the next product.

Example 1.—Multiply 6 hrs. 40 min. 17 sec. by 8.

OPERATION.

hrs.	min.	sec.
6	40	17
		8
53	22	16

Here we first multiply 17 sec. by 8 which gives us 136 sec. = 16 sec. to set down and 2 min. to carry; 40 min. × 8 = 320 min. and 2 min. carried make 322 min. = 22 min. to set down and 5 hours to carry, &c.

Example 2.—Multiply 7 lbs. 4 oz. 3 dr. 2 scr. 16 grs. by 11.

OPERATION.

lbs.	oz.	dr.	scr.	grs.
7	4	3	2	16
				11
81	1	3	0	16

Here 16 grs. × 11 = 176 grs. = 16 grs. to set down and 8 scr. to carry; 2 scr. × 11 = 22 scr. and 8 scr. carried, make 30 scr. = 0 scr. to set down and 10 drs. to carry, &c.

EXERCISE 22.

Find the value of—

1. 5 days 4 hrs. 17 min. 4 sec. × 8.
2. 6 qrs. 17 lbs. 4 oz. × 11.
3. 22 bush. 1 pk. 1 gal. 1 qt. × 6.
4. £179 14s. 11¾d. × 12.
5. 11 gal. 1 qt. 1 pt. × 11.
6. 167 lbs. 7 oz. 10 dwt. × 5.
7. 29 miles 6 fur. 17 per. × 10.
8. 164 years 11 days 17 hours × 7.
9. 46 cub. feet 319 cub. inches × 11.
10. 111 cords 7 cord ft. 7 cub. feet × 12.
11. 26 r. 16 per. 4 yds. × 8.
12. 19 cwt. 1 qr. 23 lbs. × 12.
13. £127 16s. 8¼d. × 9.
14. 111 per. 4 yds. 2 ft. 7 in. × 7.
15. 19 sq. per. 7 yds. 8 ft. × 3.
16. 179 yds. 2 qrs. 1 na. 2 in. × 11.
17. 16 lbs. 11 oz. 2 drs. 1 scr. × 10.
18. 14 qrs. 14 lbs. 11 oz. × 4.
19. 278 miles 6 fur. 11 per. × 2.
20. 64 weeks 17 hours 38 minutes × 11.
21. 17 pecks 1 gal. 1 qt. 1 pt. × 7.
22. 109 cwt. 2 qrs. 11 lbs. × 12.
23. £160 17s. 11¾d. × 9.
24. 74 a. 2 r. 7 per. 4 yds. × 9.

---

14. When the multiplier is a composite number, none of its factors being greater than 12:—

RULE.

Multiply the given multiplicand by any one factor, then multiply the resulting product by a second factor, this second product by a third, if there be any, and so on. The last product is the one sought.

EXAMPLE 1.—Multiply 7 bush. 1 gal. 1 qt. by 490.

OPERATION.

bush.	pks.	gals.	qts.
7	0	1	1
			10
71	2	0	2
			7
500	3	1	2
			7
3506	2	0	2

Here the factors of the multiplier 490, are 10 × 7 × 7, and, according to the rule, we multiply the given quantity by any one of them as 10, then the product by a second factor, and this last product by the third.

EXAMPLE 2.—Multiply 5 miles 7 fur. 19 per. 4 yds. by 72.

OPERATION.

miles	fur.	per.	yds.
5	7	19	4
			9
53	3	17	3
			8
427	3	20	2

Here the factors of the multiplier are 9 and 8. We first multiply the given multiplicand by 9, and then multiply the result by 8.

NOTE.—We might have first multiplied by 8 and then the result by 9, and we should have obtained the same result.

EXERCISE 23.

Find the value of—

1. £74 19s. 4¾d. × 16.
2. 75 lbs. 4 oz. 7 dwt. × 18.
3. 16 days 4 hours 17 min. × 21.
4. 37 Flem. ells 2 qrs. 1 na. × 35.
5. 63 miles 4 fur. 7 per. × 56.
6. 71 gals. 2 qts. 1 pt. × 77.
7. 43 hours 19 min. 36 sec. × 84.
8. 16 a. 3 r. 17 per. × 108.
9. 91 oz. 6 drs. 2 scr. 19 grs. × 121.
10. £116 11s. 11¾d. × 42.
11. 115 sq. per. 4 yds. 7 ft. × 144.
12. 93 cwt. 3 qrs. 17 lbs. × 99.

13. 16 years 110 days 11 hours × 50.
14. 29 cub. yds. 17 cub. ft. 1119 cub. in. × 48.
15. 126 bush. 1 qt. 1 pt. × 54.
16. £27 16s. 0¾d. × 100.
17. 74 per. 4 yds. 2 ft. 11 in. × 600.
18. 93 hours 17 min. 57 sec. × 1100.
19. 5 a. 2 r. 7 per. 9 yds. × 560.
20. £63 14s. 9¼d. × 8100.

17. When the multiplier is not a composite number and is greater than 12:—

RULE.

Resolve the multiplier into two or more composite numbers.

Find the product of the multiplicand by each of these separately, and add the results together for the required product.

NOTE.—If the multiplier is not greater than 100, we resolve it into *tens* and *units*; if greater than 100 and less than 1000, into *hundreds*, *tens*, and *units*; if greater than 1000 and less than 10000, into *thousands*, *hundreds*, *tens*, and *units*, &c.

Thus 89 = 80 + 9; 76 = 70 + 6; &c.
147 = 100 + 40 + 7; 747 — 700 = 40 + 7; &c.
6497 = 6000 + 400 + 90 + 7; 9162 = 9000 + 100 + 60 + 2; &c.

EXAMPLE 1.—Multiply £71 16s. 4¾d. by 79.

OPERATION.

£	s.	d.	
71	16	4¾	
		10	
718	3	11½	
		7	
5027	7	8½	= 70 times the multiplicand.
646	7	6¾	= 9 " "
5673	15	3¼	= 79 " "

Here the given multiplier, 79 70 — 9, and the factors of 70 are 10 × 7, &c.

EXAMPLE 2.—Multiply 16 cwt. 2 qrs. 17 lbs. by 867.

OPERATION.

cwt.	qrs.	lbs.		cwt.	qrs.	lbs.	
16	2	17	× 7 =	116	2	19 = 7 times mult.	
		10					
166	2	20	× 6 =	1000	0	20 = 60	"
		10					
1667	0	0	× 8 =	13336	0	0 = 800	"
			Sum =	14452	3	14 = 867	"

EXERCISE 24.

Find the value of—

1. 4 bush. 1 pk. 1 qt. × 718.
2. £16 14s. 11½d. × 867.
3. 9 days 4 hrs. 17 min. × 263.
4. 47 yds. 2 ft. 7 in. × 83.
5. 6 lbs. 4 oz. 7 dwt. × 197.
6. 7 a. 4 per. 3 ft. × 985.
7. 16 yds. 3 qrs. 1 na. × 1149.
8. 23 oz. 7 drs. 2 scr. 16 grs. × 6472.
9. £9 11s. 4¾d. × 8298.
10. 73 cwt. 1 qr. 16 lbs. × 67.
11. Multiply 7 miles 4 fur. 16 per. 2 yds. 2 ft. 6 in. by 64[illegible]
12. Multiply 17 Eng. ells 4 qrs. 2 na. 1 in. by 217.
13. Multiply 6 cwt. 1 qr. 17 lbs. 4 oz. 7 drs. by 982.
14. Multiply 8 a. 2 r. 14 per. 17 yds. 6 ft. 117 in. by 2345.
15. Multiply 11 years 217 days 23 hours 47 min. 18 sec. by [illegible]
16. Multiply 2 cords 7 cord ft. 14 cubic ft. by 103.
17. Multiply 7 bushels 1 pk. 1 gal. 1 qt. 1 pt. by 3218.
18. Multiply 67 lbs. 4 oz. 5 drs. 1 scr. 11 grs. by 975.
19. Multiply £174 16s. 0¾d. by 780.
20. Multiply 23 lbs. 11 oz. 16 dwt. 11 grs. by 359.

COMPOUND DIVISION.

18. Compound Division is the division of applicate numbers of more than one denomination.

19. Compound Division is divided into two cases.

1st. When the divisor is an abstract number.
2d. When the divisor is an applicate number.

20. When the divisor is an abstract number and not greater than 12:—

RULE.

Set the divisor to the left of the dividend.

Then, beginning at the left-hand side, divide the first term by it, put the quotient under that term, reduce the remainder, if any, to the next lower denomination, and to the number thus obtained add the given number of that lower denomination.

Divide the number thus obtained by the divisor, as before; and so on.

21. If the divisor is composite:—

RULE.

Divide, as in Rule 1, *by each factor in succession.*

22. If the divisor is not composite and is greater than 12:—

RULE.

Proceed as in Rule 1, *but write the quotient to the right of the dividend.*

Example 1.—Divide 679 lbs. 4 oz. 7 dwt. by 11.

OPERATION.

lbs.	oz.	dwt.
11)679	4	7
61	9	$2\frac{5}{11}$

Here we say 11's in 67, 6 and 1 over; 11's in 19, 1 and 8 over; 8 lbs. = 96 oz. and 4 oz. make 100 oz.; 11's in 100, 9 and 1 over; 1 oz. = 20 dwt. and 7 dwt. make 27 dwt., &c.

EXAMPLE 2.—Divide £179 16s. 9d. by 84.

OPERATION.

	£	s.	d.
7)	179	16	9
12)	25	13	9...6 rem.
	2	2	9...9 rem.

$7 \times 9 = 63 + 6 = 69$, true rem.

Then £2 2s. $9\frac{69}{84}$d. *Ans.*

Here the factors of the divisor are 7 and 12, and we divide by each, as in Example 1. From the two partial remainders we obtain the true remainder by the rule in Art. 51, Sec. II.

EXAMPLE 3.—Divide 723 yds. 2 qrs. 1 na. by 146.

OPERATION.

	yds.	qrs.	na.		yds.	qrs.	na.
146)	723	2	1	(	4	3	$3\frac{43}{146}$.
	584						
	139						
	4						
	556						
	438						
	120						
	4						
	481						
	438						
	43						

EXERCISE 25.

Find the value of—

1. £973 6s. 4d. ÷ 4.
2. 127 cwt. 2 qrs. 17 lbs. ÷ 11.
3. 172 days 16 h. 29 min. ÷ 7.
4. 614 bush. 1 pk. 1 pt. ÷ 8.
5. 4179 miles 7 fur. 9 per. ÷ 6.
6. 290 sq. per. 7 yds. 8 f ÷ 3
7. 111 lbs. 7 oz. 4 dr. 2 scr. ÷
8. 69 gals. 1 qt. 1 pt. ÷ 12.
9. £9176 14s. 9½d. ÷ 8.
10. 706 cwt. 1 qr. 16 lbs. ÷ 10.

11. £196 7s. 8d. ÷ 24.
12. 149 fur. 17 per. 4 yds. ÷ 35.
13. 1479 hrs. 47 min. 16 sec. ÷ 81.
14. 1890 lbs. 7 oz. 12 dwt. ÷ 108.
15. 679 sq. per. 7 ft. 107 in. ÷ 132.
16. 3 qrs. 19 lbs. 11 oz. 7 drs. ÷ 72.
17. 1167 yrs. 119 days 11 hrs. ÷ 144.
18. 987 oz. 7 drs. 1 scr. 16 grs. ÷ 97.
19. 1679 r. 4 per. 7 ft. 96 in. ÷ 117.
20. 7967 wks. 4 days 17 sec. ÷ 916.
21. £96749 16s. 11¼d. ÷ 113.
22. 479 c. 7 c. ft. 11 cub. ft. ÷ 89.
23. 7171° 17′ 19″ ÷ 147.
24. 1467 French ells 1 qr. 2 na. 1 in. ÷ 267.
25. 916 miles 6 fur. 4 yds. ÷ 67.
26. £1911 17s. 0¾d. ÷ 161.
27. 9134 lbs. 4 oz. 17 dwt. ÷ 963.
28. 7149 bush. 1 qt. 1 pt. ÷ 417.
29. 2716 days 14 hours 17 min. 9 sec. ÷ 603.
30. 4000 cwt. 19 lbs. 11 oz. ÷ 347.

23. When the divisor is an applicate number the quotient is an abstract number, and means *so many times*, and we proceed according to the following

RULE.

Reduce both the divisor and the dividend to the lowest denomination mentioned in either, and then proceed as in common division.

Example 1.—Divide 73 oz. 4 dwt. 17 grs. by 9 oz. 7 dwt.

OPERATION.

73 oz. 4 dwt. 17 grs. = 35153 grs.
9 oz. 7 dwt. = 4488 grs.

$$4488)35153(7\tfrac{3737}{4488}.\ \textit{Ans.}$$
$$\underline{31416}$$
$$3737$$

Here we reduce both the dividend and the divisor to grains, that being the lowest denomination contained in either. We then find that the divisor, 4488, is contained in the dividend 7 times, with a remainder, 3737, and according to the methods already explained, we set down this remainder above a line with the divisor beneath it. We may, however, read the answer, 7 *times* with a remainder of 3737 grains or 7 oz. 15 dwt. 17 grains yet to be divided.

EXAMPLE 2.—Divide £793 16s. 5¼d. by £17 14s. 9d.

OPERATION.

£793 16s. 5¼d. = 762069 farthings.
£17 14s. 9d. = 17028 "
17028)762069($44\frac{12837}{17028}$. *Ans.*
68112
———
80949
68112
———
12837

EXERCISE 26.

Find the value of—

1. 739 days 4 hours 16 min. + 23 hours 14 min. 42 sec.
2. £4967 0s. 0¼d. + £63 17s.
3. £1192 17s. 8d. + £9 17s. 4¾d.
4. 986 cwt. 2 qrs. 17 lbs. + 6 cwt. 1 qr. 17 lbs. 9 oz.
5. 426 a. 1 r. 23 per. + 2 a. 8 per. 17 yds. 4 ft.
6. 71 fur. 16 per. 3 yds. 1 ft. + 27 per. 3 yds. 2 ft. 7 in.
7. 1122 cords 3 cord ft. + 12 cords 11 cubic ft.
8. 111 lbs. 4 oz. 7 dwt. + 9 oz. 7 dwt. 17 grs.
9. 1468 Eng. ells 2 qrs. 1 na. + 73 Flem. ells 1 na. [illegible]
10. 476 bush. 1 gal. 1 pt. + 3 bush. 1 pk. 1 qt.

---

11. Divide 6 lbs. 4 oz. 1 dr. by 1 oz. 7 drs. 1 scr. 7 [illegible]
12. Divide £9 4s. 7¼d. by 3s. 11¾d.
13. Divide 7 acres by 17 sq. yds. 6 ft. 4 in.
14. Divide 927 miles 4 fur. 7 per. by 6 miles 3 inches.
15. Divide 11 years 47 days 1 hour by 23 weeks 2 days 7 hours.
16. Divide 167 bush. 1 pk. by 9 bush. 1 qt.
17. Divide 17 tons by 14 cwt. 3 qrs.
18. Divide 126 yds. 1 qr. 2 na. by 17 French ell [illegible] qr. 1 in.
19. Divide 963 miles 420 yds. by 7 fur. 63 yds.
20. Divide £1111 11s. 11¾d. by £12 13s. 4¼d.

EXERCISE 27.

Miscellaneous Problems.

1. Reduce 1769 Flemish ells to feet.
2. Add together 97 lbs. 4 oz. 7 dwt.; 16 lbs. [illegible] oz. 16 grs.; 43 lbs. 7 dwt. 9 grs.; 19 lbs. 4 oz. 11 dwt.; and 1[illegible] oz. 5 grs.

3. Express 714, 1111, 2704, 91671, 813471, and 31917169 in Roman numerals.

4. Divide £179 6s. 11½d. equally among 11 persons.

5. A sovereign weighs about 123 grains; what is the weight of £7500 in gold?

6. Read the following numbers:

1002000047006
900011110011110011
16714967904
71300400200

7. Sound travels at the rate of 1120 feet per second; the flash of a cannon, fired on one side of a river, is observed by a person standing directly opposite on the other side 11 seconds before he hears the report. How many miles, fur., &c., is the river in width?

8. The new Canadian cent is exactly 1 inch in diameter and 100 weigh exactly 1 lb. Avoirdupois; what would be the weight and worth of that number of cents which would reach completely round the earth, the circumference being 24902¼ miles?

9. How long would it require to count $794671 in twenty-cent pieces, at the rate of 108 coins per minute?

10. If a person spends upon an average $2·17 per day, how much does he spend during the year?

11. What is the weight of 3 dozen silver forks, each weighing 4 oz. 1 dwt. 6 grs.?

12. Bought 1 lb. of tea for 75 cents, 3 lbs. of coffee at 14 cents per lb., 6 lbs. of rice at 5 cents per lb., 27 lbs. of sugar at 11 cents per lb., 13 lbs. of raisins at 15 cents per lb., and a barrel of flour for $7·20; how much have I to pay for the whole?

13. Write the following expressions in common figures: XI, $\overline{\text{LD}}$, MMMCCCXXXIII, $\overline{\text{MMDCLX}}\text{C}$, $\text{L}\overline{\text{XXX}}\text{M}\overline{\text{XX}}\text{LIV}$, $\overline{\overline{\text{CDL}}}\text{MDCCIX}$, $\overline{\text{MXC}}\text{M}\overline{\text{V}}$, and $\overline{\overline{\text{M}}\text{M}\overline{\overline{\text{V}}}\text{C}}\text{MDCCII}$.

14. How many times can 167 be subtracted from 271496?

15. When the multiplier is 714 and the multiplicand 9167, what is the product?

16. What is the ninth part of 67 a. 4 per. 17 yds.?

17. Divide £16 11s. among 3 persons, so that one shall have £4 2s. more than each of the others.

18. Divide $744 among four persons, so that the first shall have one-sixth of the whole, the second one-fourth of the remainder, and the other two, each half of what then remains. What is the share of each?

19. If A has £176 4s. 5¾d. and B has $694·70; which has most, and how much?

20. A regiment of soldiers contains 1147 men; how much cloth would it require to make coats for the whole, each coat taking 4 yds. 1 qr. 3 na.?

21. What is the weight of $7196·40 in cent pieces, Canadian money?

22. Reduce 7 miles 4 fur. 17 per. to fathoms.

23. The quotient is 749, the divisor 47; what is the dividend?

24. What is the difference between $\overline{\text{X}}$MMCI and 16701?

25. The minuend is 71467, the remainder 61794, what is the subtrahend?

26. Divide $679 among two persons, so that the first shall have $146 more than the second. What is the share of each?

27. What is the product of 714 + 16 + 179 + 42 + 93, multiplied by 91467 − 234 − 946 − 1127 − 80040 + 27 − 67 + 83?

28. How many bushels of wheat are there in 71496 lbs.?

29. Write down as one number, six trillions seven millions ninety-six thousand four hundred and five.

30. The sum of two numbers is 1746; one is 974, what is the other?

31. What is the cost of 23 pair of shoes at 6s. 11d. per pair?

32. A gallon of water weighs 10 lbs. and a cubic foot weighs 62½ lbs.; how many gallons are there in 748 cubic feet?

33. Two men, A and B, run a race. A gives B a start of 17 yards, but gains on him at the rate of 2 feet in 5 yards; how much will A be in advance of B when B has run one mile?

34. Divide $749·60 among A, B and C, so that A shall have as much as B and C together, and B and C equal shares. What is the share of each?

35. 2366 cubic feet of wood are to be divided among three charitable institutions, so that as often as the first receives 2 cubic feet the second shall receive 5 and the third 7; how many cords does each receive?

36. A farmer owned 247 acres of land and disposed of it as follows: he gave 1 a. 1 r. 17 per. for a school site, sold 17 a. 23 per., gave 21 a. 1 r. to his wife, and divided the remainder equally among his 3 sons; how much did each son receive?

37. If 17 seconds elapse between the flash of lightning and the arrival of the report; allowing sound to travel at the rate of 1120 feet per second, how far off is the thunder-cloud?

38. The London Times has a circulation of 12000 per day; if it be sold at 5d. per copy, express in pounds, shillings, and pence, and also in dollars and cents, the sum realized by its sale for one entire year (313 days).

39. The greater of two numbers is 710 and their difference 297, what is the smaller number?

40. The Jewish shekel weighed 210 grains Troy, and was worth about 2s. 9½d. Canadian currency; what was the weight of a talent, containing 3000 shekels, and what was the value of 500 talents in dollars and cents?

41. A wished to exchange 297 yards of cloth at £1 7s. 4½d. per yard with B for flour at $3·17 per barrel; how many barrels of flour should he receive?

SECTION III.

GREATEST COMMON MEASURE AND LEAST COMMON MULTIPLE.

GREATEST COMMON MEASURE.

1. A Measure of a number is any number that will exactly divide it; that is, leaving no remainder.

2. A Common Measure of two or more numbers is any number that will *exactly* divide each of them.

3. The Greatest Common Measure of two or more numbers is the *greatest* number that exactly divides each of them.

Thus 2, 3, 4, 6, 8, 12, and 24 are all common measures of 24 and 48, but 24 alone is the *greatest* common measure, because it is the greatest number that divides both 24 and 48 without a remainder.

NOTE.—The Greatest Common Measure is usually indicated by the initial letters G. C. M.

4. To find the G. C. M. of two numbers :—

RULE.

Divide the greater of the two given numbers by the smaller, then the divisor by the remainder; next the last divisor by the new remainder; and so on until there is no remainder. The last divisor will be the G. C. M. required.

NOTE.—If the last divisor is unity, the given numbers have no common measure; i. e., they are prime to one another.

EXAMPLE 1.—What is the G. C. M. of 1825 and 2555?

OPERATION.

```
1825)2555(1
     1825
     ----
      730)1825(2
          1460
          ----
           365)730(2
               730
               ---
```

Here we divide the greater number, 2555, by the less, 1825, and thus obtain a remainder, 730, which we now make the divisor, and 1825, the former divisor, becomes the dividend. We find that 730 goes into 1825 twice, and gives a remainder, 365; and so on. When

365 is used, it leaves no remainder, and is therefore the G. C. M.

Example 2.—What is the G. C. M. of 647 and 2750?

OPERATION.

```
647)2750(4
    2588
    ----
     162)647(3
         486
         ---
         161)162(1
             161
             ---
               1)161(161
                 161
                 ---
```

Here, in following the rule, we find that the first divisor that will go into the *then* dividend is 1; or, in other words, the numbers have no common measure.

Exercise 28.

Find the G. C. M. of the following numbers:

1. 1024 and 2240.
2. 1902 and 24409.
3. 1624 and 14500.
4. 8393 and 4609.
5. 714 and 1176.
6. 219 and 11476.
7. 194706 and 289913.
8. 2925 and 29484.
9. 27525 and 1725.
10. 2254 and 71001.
11. 11256 and 198[illegible].
12. 5161 and 7755
13. 87147 and 178[illegible]
14. 1261 and 663.
15. 918 and 1347.
16. 187 and 265.
17. 1914 and 357[illegible]6.
18. 21671 and 22[illegible]11.
19. 82159 and 58[illegible]
20. 452 and 212.

LEAST COMMON MULTIPLE.

5. One number is said to be a multiple of another when it exactly contains, as divisor, that other a certain number of times.

6. A Common Multiple of two or more numbers is any number that exactly contains each of them as divisor.

7. The Least Common Multiple of two or more numbers is the *least* number that exactly contains each of them as divisor.

Thus, 6, 12, 18, 24, 30, 36, &c., are all common multiples of 2 and 3, but the *least* common multiple of 2 and 3 is 6.

NOTE.—The Least Common Multiple of two or more numbers is usually represented by the initial letters l. c. m.

8. To find the l. c. m. of two or more numbers:—

RULE.

Set down the given numbers in a line, and strike out any that are exactly contained in any of the others.

Take any one of the uncancelled numbers as divisor, set it to the left of the line, and strike out of each remaining number the highest factor that is common to it and the assumed divisor.

Write the uncancelled factors and numbers in a line, and again strike out any that are exactly contained in any of the others. Assume any one of the uncancelled numbers in this second line as divisor, and proceed with it as before.

Proceed thus until no two numbers left in the new line have any common measure but unity.

Then, to get the l. c m., multiply together all the assumed divisors and all the numbers left in the last line.

EXAMPLE 1.—What is the l. c. m. of 6, 8, 10, 12, 15, 16, 20, 24, and 30?

OPERATION.

$$\begin{array}{r|r} 20) & \cancel{6}..\cancel{8}..\cancel{10}..\cancel{12}..\cancel{15}..16..20..24..30 \\ \hline 4) & 4.. \quad ..\ 6..\ \cancel{3} \\ \hline & 3 \quad\quad\quad\quad \end{array}$$

l. c. m. = 20 × 4 × 3 = 240.

Here, after arranging the given numbers, we strike out 6, 8, and 12, because they are exactly contained in 24, another of the given numbers. For a similar reason, we strike out 10 and 15 (i. e. because they are exactly contained in 30). Next we assume 20, one of the remaining numbers, as divisor. The highest factor of 16 that is also a factor of the assumed number 20 is 4, and we accordingly divide 16 by 4, and for a similar reason we divide 24 by 4 and 30 by 10. The uncancelled factors are now 4, 6, and 3, of which

3 is exactly contained in 6, and we therefore strike it out. This leaves 4 and 6, of which we assume 4 as divisor, and, as this contains 2, a factor of 6, we divide the 6 by 2. Then, multiplying the 3, remaining, by the assumed divisors 4 and 10, we get 240 for the l. c. m.

Example 2.—What is the l. c. m. of 16, 24, 28, 30, 32, 36, 40, 44, 45, 48, and 50?

OPERATION.

$$\begin{array}{l} 40)\underline{\cancel{16}..\cancel{24}..28..30..32..36..40..44..45..48..50} \\ \qquad\qquad\qquad 6)\underline{7..\ \cancel{3}..\ 4..\ \cancel{9}..\quad ..11..\ 9..\ 6..\ 5} \\ \qquad\qquad\qquad\quad 7..\quad ..\ 2..\quad ..\quad ..11..\ 3..\quad ..\ 5 \end{array}$$

Then l. c. m. $= 40 \times 6 \times 7 \times 2 \times 11 \times 3 \times 5 = 554400$.

Here we strike out at once 16 and 24, since they are contained exactly in 48. Then we assume 40 as divisor, of which 4, one of its factors, reduces 28 to 7, 36 to 9, and 44 to 11. Also 8, another factor, reduces 32 to 4 and 48 to 6. Also 10, another factor, reduces 30 to 3 and 50 to 5. Also 5, another factor, reduces 45 to 9. Next we strike out 3 and 9 in the second line, since they are each contained in 9, another number in that line, &c.

Exercise 29.

Find the l. c. m. of—

1. 6, 9, and 30.
2. 30 and 55.
3. 7, 21, 35, 4, and 20.
4. 2, 9, 16, 35, 56, and 63.
5. 2, 4, 6, 8, 10, 12, 16, 18, and 20.
6. 8, 9, 11, 22, 72, 82, and 99.
7. 6, 10, 14, 18, 22, 28, and 32.
8. 5, 10, 15, 20, 25, 30, 35, and 40.
9. 1, 2, 3, 4, 5, 6, 7, 8, and 9.
10. 3, 6, 9, 12, 48, 21, 24, and 16.
11. 8, 21, 63, 40, 160, 240, and 300.
12. 16, 41, and 38.
13. 9 and 16.
14. 112, 200, and 72.
15. 90, 36, 63, 12, and 7.
16. 3, 5, 7, 9, and 11.
17. 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, and 32.
18. 25, 7, 44, 60, 63, 55, 9, 11, 28, 70, and 4.
19. 720, 396, 252, and 540.
20. 15, 12, 128, 30, 16, 4, 320, and 96.

SECTION IV.

VULGAR AND DECIMAL FRACTIONS.

VULGAR FRACTIONS.

1. A Fraction is an expression representing one or more of the equal parts into which any quantity may be divided.

One third is written......$\frac{1}{3}$
One fifth is written.......$\frac{1}{5}$
One seventh is written...$\frac{1}{7}$
Six sevenths is written...$\frac{6}{7}$
Nineteen twenty-sevenths is written............$\frac{19}{27}$

Four ninety-eighths is written.............. $\frac{4}{98}$
Seven hundred eleven-hundredths is written.................... $\frac{700}{1100}$
&c.

2. If a quantity be divided into 3, 5, 7, 11, &c., equal parts, then one of these equal parts is called *one third*, *one fifth*, *one seventh*, *one eleventh*, &c., as the case may be.

3. Every fraction is expressed by two numbers, called *terms*, written one above the other and separated by a line.

4. The number written below the line is called the *Denominator*, because it shows the denomination, i. e., tells into how many equal parts the quantity is supposed to be divided.

5. The number above the line is called the *Numerator*, because it numerates or tells how many of these parts are to be taken.

6. Every fraction expresses the division of the numerator by the denominator, and the little horizontal line which separates the two terms is derived from and stands for the sign of division.

Thus, $\frac{2}{7}$ means either the $\frac{1}{7}$ part of 2 or 2 times the $\frac{1}{7}$ part of 1. $\frac{13}{34}$ means either the $\frac{1}{34}$ part of 13 or 13 times the $\frac{1}{34}$ part of 1, &c.

7. Since every fraction expresses the division of the numerator by the denominator, it follows that—

The *value* of the fraction is the *quotient* obtained by dividing the numerator by the denominator, and hence,

Multiplying the numerator of a fraction by any number, multiplies the fraction by that number.

Multiplying the denominator of a fraction by any number, divides the fraction by that number.

Multiplying both numerator and denominator of a fraction by the same number, does not affect the value of the fraction.

Dividing the numerator of a fraction by any number, divides the fraction by that number.

Dividing the denominator of a fraction by any number, multiplies the fraction by that number.

Dividing both numerator and denominator of a fraction by the same number, does not affect its value.

8. Fractions are divided into two classes, Vulgar and Decimal.

9. A Decimal Fraction is a fraction in which the denominator is 1, followed by one or more 0's.

10. All other fractions are Vulgar or Common Fractions.

NOTE.—The word "vulgar" is here used in the sense of "common."

11. There are six kinds of Vulgar Fractions—*Proper*, *Improper*, *Mixed*, *Simple*, *Compound*, and *Complex*.

12. A Proper Fraction is one in which the denominator is greater than the numerator.

A Proper Fraction may also be defined to be a fraction whose value is less than 1.

Thus [illegible], &c., are proper fractions.

13. An Improper Fraction is a fraction whose denominator is *not* greater than its numerator.

An Improper Fraction may also be defined to be a fraction whose value is equal to or greater than 1.

Thus [illegible], &c., are improper fractions.

14. A Mixed Number is a number made up of a whole number and a fraction.

Thus $16\frac{3}{8}$, $193\frac{4}{7}$, $1\frac{13}{17}$, $999\frac{3}{7}$, $6\frac{3}{11}$, $2\frac{1}{3}$, &c., are mixed numbers.

15. A Simple Fraction expresses one or more equal parts of unity.

Thus $\frac{4}{7}$, $\frac{9}{8}$, $\frac{8}{9}$, $\frac{11}{17}$, $\frac{4}{5}$, $\frac{163}{264}$, &c., are simple fractions.

16. A Compound Fraction expresses one or more equal parts of a fraction, or, in other words, is a fraction of a fraction.

Thus $\frac{2}{3}$ of $\frac{3}{4}$, $\frac{4}{5}$ of $\frac{7}{9}$ of $\frac{11}{13}$ of $\frac{6}{5}$ of $\frac{139}{5}$, &c., are compound fractions.

17. A Complex Fraction has a fraction or a mixed number in its numerator or in its denominator, or both.

Thus $\frac{4}{3\frac{1}{2}}$, $\frac{16\frac{1}{4}}{7}$, $\frac{3\frac{5}{7}}{9\frac{3}{11}}$, $\frac{\frac{1}{7}}{9}$, $\frac{10}{\frac{1}{3}}$, &c., are complex fractions.

18. Any whole number may be made a fraction by placing 1 beneath it for denominator.

Thus $5 = \frac{5}{1}$, $17 = \frac{17}{1}$, $11 = \frac{11}{1}$, $217 = \frac{217}{1}$, &c.

EXERCISE.

1. Read the following fractions: $\frac{7}{16}$, $6\frac{203}{407}$, $\frac{1127}{634}$, $\frac{913}{16}$, $\frac{42}{1080}$, $\frac{71}{207}$.
2. Read the following fractions: $7\frac{1}{4}$, $11\frac{3}{11}$, $\frac{9}{8}$, $217\frac{4}{19}$, $603\frac{47}{100}$, $11379\frac{6047}{98716}$.
3. Read the following fractions: $\frac{7}{7}$, $\frac{407}{90}$, $\frac{116}{113}$, $\frac{43}{27}$, $9\frac{43}{1016}$, $\frac{31}{56}$, $\frac{1003}{1093}$.
4. Read all the *proper fractions* found in the above.
5. Read all the *improper fractions*.
6. Read all the *mixed numbers*.
7. Write down on your slate any six proper fractions.
8. Write down on your slate any six improper fractions.

9. Write down on your slate any six mixed numbers.
10. Write down on your slate any six decimal fractions.
11. Write down on your slate any six simple fractions.
12. Write down on your slate any six compound fractions.
13. Write down on your slate any six complex fractions.
14. Express 7, 9, 4, 23, 17, 34, 109, and 207 as fractions.

REDUCTION OF VULGAR FRACTIONS.

19. To reduce an integral number to a fraction having a given denominator:—

RULE.

Write the integral number as a fraction having 1 *for denominator, and multiply both numerator and denominator of the resulting expression by the given denominator.*

Example 1.—Reduce 123 to a fraction having 20 for denominator.

OPERATION.

$$123 = \frac{123}{1}, \text{ and } \frac{123 \times 20}{1 \times 20} = \frac{2460}{20} \textit{ Ans.}$$

Example 2.—Reduce 17 to a fraction having 29 for denominator.

OPERATION.

$$17 = \frac{17}{1}, \text{ and } \frac{17 \times 29}{1 \times 29} = \frac{493}{29} \textit{ Ans.}$$

Exercise 30.

1. Reduce 7, 9, 27 and 40 to fractions having 11 for denominator.
2. Reduce 2, 207, 440, and 9 to fractions having 100 for denominator.
3. Reduce 22, 47, 69, and 100 to fractions having 93 for denominator.

4. Reduce 217, 613, 927, and 4 to fractions having 13 for denominator.

5. Reduce 27, 304, 617, and 93 to fractions having 248 for denominator.

6. Reduce 209, 407, 789, and 5 to fractions having 611 for denominator.

---

20. To reduce a mixed number to an improper fraction:—

RULE.

Multiply the whole number by the denominator of the fraction, to the product add the given numerator and place the sum over the given denominator.

EXAMPLE 1.—Reduce $7\frac{4}{9}$ to an improper fraction.

OPERATION.

$$\begin{array}{r} 7\frac{4}{9} \\ 9 \\ \hline \frac{67}{9} \end{array}$$

Here we multiply the whole number, 7, by the denominator, 9, and to the product, 63, add the numerator, 4. This gives 67 for the numerator, beneath which we write the given denominator, 9.

EXAMPLE 2.—Reduce $167\frac{14}{19}$ to an improper fraction.

OPERATION.

$$167 \times 19 = 3173 \text{ and } 3173 + 14 = 3187;$$
hence $167\frac{14}{19} = \frac{3187}{19}$. *Ans.*

EXERCISE 31.

Reduce the following mixed numbers to improper fractions:

1. $16\frac{3}{7}$.
2. $9\frac{2}{11}$.
3. $14\frac{3}{10}$.
4. $71\frac{3}{5}$.
5. $161\frac{21}{40}$.
6. $27\frac{19}{31}$.
7. $186\frac{14}{27}$.
8. $200\frac{1}{200}$.
9. $713\frac{410}{617}$.
10. $91\frac{11}{17}$.
11. $6\frac{4}{13}$.
12. $2097\frac{4}{11}$.
13. $617\frac{123}{929}$.
14. $417\frac{17}{18}$.
15. $6170\frac{181}{1000}$.
16. $111\frac{111}{1111}$.
17. $214\frac{11}{107}$.
18. $63\frac{19}{20}$.
19. $2345\frac{345}{647}$.
20. $1919\frac{19}{191}$.

---

21. To reduce an improper fraction to its equivalent mixed number:—

RULE.

Divide the numerator by the denominator, and the quotient will be the required mixed number.

EXAMPLE 1.—Reduce $\frac{1714}{9}$ to a mixed number.

OPERATION.

$$\frac{1714}{9} = 1714 \div 9 = 190\tfrac{4}{9}.\ \textit{Ans.}$$

EXAMPLE 2.—Reduce $\frac{14716}{109}$ to a mixed number.

OPERATION.

$$\frac{14716}{109} = 14716 \div 109 = 135\tfrac{1}{109}.\ \textit{Ans.}$$

EXERCISE 32.

Reduce the following improper fractions to their equivalent mixed numbers:

1. $\frac{71}{4}$.
2. $\frac{1694}{11}$.
3. $\frac{21794}{17}$.
4. $\frac{2826262}{43}$.
5. $\frac{714}{19}$.
6. $\frac{200}{7}$.
7. $\frac{11124}{643}$.
8. $\frac{213451}{1632}$.
9. $\frac{2179}{87}$.
10. $\frac{14627}{93}$.
11. $\frac{127963}{122}$.
12. $\frac{113}{17}$.
13. $\frac{4547}{48}$.
14. $\frac{123}{13}$.
15. $\frac{279}{61}$.
16. $\frac{417896}{6139}$.
17. $\frac{61427}{23}$.
18. $\frac{11347}{92}$.
19. $\frac{91347}{834}$.
20. $\frac{12345}{6431}$.

---

22. To reduce a fraction to its lowest terms:—

RULE.

Divide both numerator and denominator by their greatest common measure.

EXAMPLE 1.—Reduce $\frac{1512}{2376}$ to its lowest terms.

OPERATION.

By Art. 4, Sec. III., the G. C. M. of 1512 and 2376 is 216. Then $1512 \div 216 = 7$, and $2376 \div 216 = 11$; hence $\frac{1512}{2376} = \frac{7}{11}$. *Ans.*

EXAMPLE 2.—Reduce $\frac{481}{26377}$ to its lowest terms.

OPERATION.

The G. C. M. of 481 and 26377 is 13.

Then $481 \div 13 = 37$, and $26377 \div 13 = 2029$; hence $\frac{481}{26377} = \frac{37}{2029}$. *Ans.*

EXERCISE 33.

Reduce the following fractions to their lowest terms :

1. $\frac{1540}{3740}$.	4. $\frac{6417}{7659}$.	7. $\frac{3073}{7091}$.	10. $\frac{1597}{701}$.
2. $\frac{4764}{6546}$.	5. $\frac{3902}{118}$.	8. $\frac{36579}{39867}$.	11. $\frac{58469}{111983}$.
3. $\frac{3307}{3331}$.	6. $\frac{2737}{406433}$.	9. $\frac{9917}{6119}$.	12. $\frac{49591}{27169}$.

NOTE.—A fraction can sometimes be reduced to its lowest terms, and the work may almost always be materially lessened, by dividing both numerator and denominator by *any* number which will go into each of them without a remainder. In order to facilitate this mode of reduction, it is necessary to remember the following facts :

1st. Any number that ends in 5 is divisible by 5.
2d. Any number that ends in 0 is divisible by 10, 5, or 2.
3d. Any number that ends in an even number is divisible by 2.
4th. When the two right-hand figures are divisible by 4, the whole is divisible by 4.
5th. When the three right-hand figures are divisible by 8, the whole number is divisible by 8.
6th. When the sum of the digits of a number is divisible by 9, the sum itself is divisible by 9 or by 3.
7th. When the sum of the digits of a number is divisible by 3, the number itself is divisible by 3.
8th. When the sum of the digits standing in the *even* places is equal to the sum of the digits standing in the *odd* places, the number is divisible by 11.

Thus the number 7416 is divisible by 4, because 16 (the last two digits) is divisible by 4.

——is divisible by 8, because 416 (its last three digits) is divisible by 8.

——is divisible by 9, because the sum of its digits $(7 + 4 + 1 + 6 = 18)$ is divisible by 9.

——is divisible by 3, because the sum of its digits $(7 + 4 + 1 + 6 = 18)$ is divisible by 3.

So also the number 4567321 is divisible by 11, since the sum of the digits in the odd places, $1 + 3 + 6 + 4 = 14 = 2 + 7 + 5$: the sum of the digits in the even places.

---

23. To reduce two or more fractions to equivalent fractions having a common denominator :—

RULE.

Find the least common multiple of all the denominators. Multiply both terms of each fraction by the quotient obtained by dividing this least common multiple by the denominator of the fraction.

Example 1.—Reduce $\frac{1}{4}$, $\frac{3}{7}$, $\frac{4}{9}$, $\frac{2}{8}$, $\frac{3}{2}$, and $\frac{2}{3}$, to equivalent fractions having a common denominator.

OPERATION.

The least common multiple of the denominators, 4, 7, 9, 8, 2, and 3, is 504.

Then $504 \div 4 = 126$, the multiplier for both terms of the 1st fraction.

$504 \div 7 = 72$, the multiplier for both terms of the 2d fraction.

$504 \div 9 = 56$, the multiplier for both terms of the 3d fraction.

$504 \div 8 = 63$, the multiplier for both terms of the 4th fraction.

$504 \div 2 = 252$, the multiplier for both terms of the 5th fraction.

$504 \div 3 = 168$, the multiplier for both terms of the 6th fraction.

And the fractions are $\frac{126}{504}$, $\frac{216}{504}$, $\frac{224}{504}$, $\frac{126}{504}$, $\frac{756}{504}$, and $\frac{336}{504}$.

Example 2.—Reduce $\frac{4}{7}$, $\frac{3}{5}$, $\frac{4}{9}$, and $\frac{7}{10}$, to equivalent fractions having a common denominator.

OPERATION.

The l. c. m. of 7, 5, 9, and 10 is 630.

Multiply both terms of the 1st fraction by 90 (i. e. $\frac{630}{7}$),
" " 2d " by 126 (i. e. $\frac{630}{5}$),
" " 3d " by 70 (i. e. $\frac{630}{9}$),
" " 4th " by 63 (i. e. $\frac{630}{10}$),
and the fractions become $\frac{360}{630}$, $\frac{378}{630}$, $\frac{280}{630}$, and $\frac{441}{630}$.

Exercise 34.

Reduce to equivalent fractions having a common denominator.

1. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{7}{10}$.
2. $\frac{2}{5}$, $\frac{3}{7}$, $\frac{4}{9}$, $\frac{2}{3}$, and $\frac{1}{4}$.
3. $\frac{7}{18}$, $\frac{5}{7}$, $\frac{5}{9}$, $\frac{5}{6}$, and $\frac{3}{14}$.
4. $\frac{7}{24}$, $\frac{11}{12}$, $\frac{3}{8}$, $\frac{2}{3}$, $\frac{7}{10}$, and $\frac{13}{18}$.
5. $\frac{17}{36}$, $\frac{9}{10}$, $\frac{4}{15}$, $\frac{23}{36}$, $\frac{5}{6}$, and $\frac{1}{6}$.
6. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{10}$.
7. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$.
8. $\frac{7}{8}$, $\frac{8}{9}$, $\frac{9}{10}$, $\frac{11}{12}$, $\frac{13}{18}$, and $\frac{17}{18}$.
9. $\frac{11}{13}$, $\frac{14}{17}$, and $\frac{16}{19}$.
10. $\frac{12}{17}$, $\frac{9}{16}$, $\frac{15}{16}$, and $\frac{7}{12}$.
11. $\frac{9}{10}$, $\frac{11}{12}$, $\frac{13}{18}$, and $\frac{19}{21}$.
12. $\frac{2}{5}$, $\frac{4}{7}$, $\frac{6}{11}$, $\frac{4}{9}$, $\frac{2}{3}$, $\frac{11}{14}$, and $\frac{13}{21}$.

24. To reduce a compound fraction to a simple one :—

RULE.

Multiply all the numerators together for a new numerator, and all the denominators together for a new denominator.

NOTE.—Before applying this rule, we may cast out or cancel all the factors that are common to a numerator and a denominator of the compound fraction.

EXAMPLE 1.—Reduce $\frac{6}{11}$ of $\frac{4}{7}$ of $\frac{3}{5}$ of $\frac{22}{27}$ of $\frac{35}{16}$ to a simple fraction.

STATEMENT.

$$\frac{6}{11} \text{ of } \frac{4}{7} \text{ of } \frac{3}{5} \text{ of } \frac{22}{27} \text{ of } \frac{35}{16} = \frac{6 \times 4 \times 3 \times 22 \times 35}{11 \times 7 \times 5 \times 27 \times 16}$$

CANCELLED.

$$= \frac{\overset{\cancel{2}}{\cancel{6}} \times \cancel{4} \times \cancel{3} \times \overset{\cancel{2}}{\cancel{22}} \times \overset{\cancel{5}}{\cancel{35}}}{\cancel{11} \times \cancel{7} \times \cancel{5} \times \underset{\underset{3}{\cancel{9}}}{\cancel{27}} \times \underset{\underset{\cancel{4}}{\cancel{8}}}{\cancel{16}}} = \frac{1}{3}. \quad \textit{Ans.}$$

Here 6 and 27 contain a common factor, 3, which is cast out, and these numbers thus reduced to 2 and 9. Next, this 2 reduces 16 to 8, and the 9 is reduced to 3 by the third numerator, which is thus cancelled. Again, 11 cancels 11 (the first denominator) and reduces 22 to 2, and this 2 reduces the 8, before obtained from the 16, to 4. Next, this 4 is cancelled by the 4 in the numerator. Again, 7 cancels the 7 in the denominator and reduces the 35 in the numerator to 5, and this 5 cancels the 5 in the denominator. All the numerators are now reduced to unity, as also all the denominators but the fourth, which is 3. The resulting fraction is therefore $\frac{1 \times 1 \times 1 \times 1 \times 1}{1 \times 1 \times 1 \times 3 \times 1}$; but as 1 is never considered as a multiplier or divisor, we write the result simply as $\frac{1}{3}$.

Example 2.—Reduce $\frac{7}{11}$ of $\frac{4}{6}$ of $\frac{3}{5}$ of $\frac{55}{20}$ to a simple fraction.

STATEMENT.

$$\frac{7}{11} \text{ of } \frac{4}{6} \text{ of } \frac{3}{5} \text{ of } \frac{55}{20} = \frac{7 \times 4 \times 3 \times 55}{11 \times 6 \times 5 \times 20}$$

CANCELLED.

$$= \frac{7 \times \cancel{4} \times \cancel{3} \times \overset{5}{\cancel{55}}}{\cancel{11} \times \underset{2}{\cancel{6}} \times \cancel{5} \times \underset{5}{\cancel{20}}} = \frac{7}{2 \times 5} = \frac{7}{10}. \quad \textit{Ans.}$$

Note 2.—If any term of the compound fraction be a mixed number, it must be reduced to its equivalent improper fraction before applying the rule.

Exercise 35.

Reduce to equivalent simple fractions.

1. $\frac{1}{3}$ of $\frac{4}{9}$ of $\frac{27}{40}$ of $\frac{10}{13}$ of $\frac{7}{8}$.
2. $\frac{1}{2}$ of $\frac{1}{8}$ of $\frac{4}{5}$ of $\frac{15}{17}$ of $\frac{34}{19}$.
3. $\frac{3}{7}$ of $\frac{1}{4}$ of $\frac{14}{51}$ of $\frac{9}{16}$ of 32.
4. $2\frac{3}{7}$ of $\frac{1}{9}$ of $\frac{3}{34}$ of $\frac{9}{11}$ of 22.
5. $\frac{2}{5}$ of $\frac{4}{11}$ of $27\frac{1}{2}$ of $\frac{1}{4}$.
6. $\frac{2}{11}$ of $3\frac{1}{9}$ of $48\frac{1}{2}$ of $\frac{1}{31}$.
7. $6\frac{3}{4}$ of $9\frac{1}{7}$ of $\frac{1}{8}$ of $\frac{7}{9}$ of $5\frac{2}{7}$.
8. $27\frac{1}{4}$ of $\frac{1}{9}$ of $\frac{11}{17}$ of $\frac{153}{225}$.
9. $\frac{2}{11}$ of $8\frac{3}{4}$ of $6\frac{3}{7}$ of $\frac{1}{26}$.
10. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{8}{11}$.
11. $2\frac{1}{4}$ of $3\frac{1}{3}$ of $4\frac{1}{3}$ of $5\frac{1}{4}$ of $6\frac{1}{5}$.
12. $3\frac{3}{7}$ of $8\frac{3}{4}$ of $9\frac{2}{7}$ of $\frac{11}{13}$ of $\frac{16}{23}$ of $\frac{1}{26}$.

25. To reduce a complex fraction to a simple one:—

RULE.

Reduce both numerator and denominator to simple fractions.

Then multiply the extremes or outside numbers together for a new numerator, and the means or intermediate numbers together for a new denominator.

EXAMPLE 1.—Reduce $\frac{4\frac{1}{2}}{\frac{7}{11}}$ to a simple fraction.

$$\frac{4\frac{1}{2}}{\frac{7}{11}} = \frac{\frac{9}{2}}{\frac{7}{11}} = \frac{9 \times 11}{2 \times 7} = \frac{99}{14} = 7\tfrac{1}{14}. \quad \textit{Ans.}$$

NOTE.—Factors that are common to one of the extremes and one of the means, are to be struck out or cancelled.

EXAMPLE 2.—Reduce $\frac{7\frac{4}{11}}{1\frac{13}{77}}$ to a simple fraction.

$$\frac{7\frac{4}{11}}{1\frac{13}{77}} = \frac{\frac{81}{11}}{\frac{90}{77}} = \frac{\overset{9}{\cancel{81}} \times \overset{7}{\cancel{77}}}{\cancel{11} \times \underset{10}{\cancel{90}}} = \frac{7 \times 9}{10} = \frac{63}{10} = 6\tfrac{3}{10}. \quad \textit{Ans.}$$

EXERCISE 36.

Reduce to equivalent simple fractions:

1. $\frac{2}{\frac{3}{5}}$

2. $\frac{\frac{7}{9}}{11}$

3. $\frac{\frac{4}{7}}{\frac{9}{11}}$

4. $\frac{2\frac{3}{4}}{7}$

5. $\frac{9}{3\frac{3}{11}}$

6. $\frac{2\frac{1}{5}}{19\frac{3}{7}}$

7. $\frac{6\frac{4}{5}}{7\frac{2}{9}}$

8. $\frac{4\frac{3}{7}}{9\frac{1}{2}}$

9. $\frac{\frac{6\frac{1}{2}}{9}}{7\frac{1}{2}}$

10. $\frac{4\frac{2}{9}}{\frac{7\frac{1}{2}}{3\frac{1}{4}}}$

11. $\frac{2\frac{7}{11}}{4\frac{1}{8}}$

12. $\cfrac{\frac{6\frac{2}{5}}{3\frac{1}{2}}}{\cfrac{7\frac{1}{2}}{\frac{9\frac{1}{4}}{7}}}$

26. To reduce a denominate fraction from one denomination to another:—

RULE.

If the reduction be from a lower to a higher denomination multiply the denominator, but if from a higher to a lower denomination multiply the numerator, as in reduction of whole numbers.

Example 1.—Reduce $\frac{7}{17}$ of an hour to the fraction of a week.

OPERATION.

$$\tfrac{7}{17}\text{ h.} = \frac{7}{17\times24}\text{ d.} = \frac{7}{17\times24\times7}\text{ wk.} = \frac{1}{17\times24} = \frac{1}{408}.\quad \textit{Ans.}$$

Or briefly,

$$\tfrac{7}{17}\text{ h.} = \frac{7}{17\times24\times7} = \frac{1}{408}\text{ of a week.}\quad \textit{Ans.}$$

Example 2.—Reduce $\frac{1}{7}$ of $\frac{3}{5}$ of $\frac{4}{9}$ of 35 oz. to the fraction of a pound Avoir.

OPERATION.

$$\tfrac{1}{7}\text{ of }\tfrac{3}{5}\text{ of }\tfrac{4}{9}\text{ of 35 oz.} = \tfrac{4}{3}\text{ oz. (by Art. 24.)}$$

$$\text{Then }\tfrac{4}{3}\text{ oz.} = \frac{4}{3\times16} = \tfrac{1}{12}\text{ lb.}\quad \textit{Ans.}$$

Example 3.—Reduce $\frac{7}{5}$ of an acre to the fraction of a yard.

OPERATION.

$$\tfrac{7}{5}\text{ of an acre} = \frac{7\times4\times40\times30\frac{1}{4}}{5}\text{ of a yard}$$

$$= \frac{33880}{5}\text{ of a yard} = \frac{6776}{1}.\quad \textit{Ans.}$$

Example 4.—Reduce $\frac{5}{7}$ of $\frac{4}{11}$ of $\frac{33}{35}$ of $\frac{48}{60}$ of 25 furlongs to the fraction of $\frac{3}{5}$ of $\frac{3}{5}$ of $\frac{5}{9}$ of 7 feet.

OPERATION.

$$\tfrac{5}{7}\text{ of }\tfrac{4}{11}\text{ of }\tfrac{33}{35}\text{ of }\tfrac{48}{60}\text{ of 25 furlongs} = \tfrac{4}{3}\text{ of a fur.}$$

$$\tfrac{3}{5}\text{ of }\tfrac{3}{5}\text{ of }\tfrac{5}{9}\text{ of 7 feet} = \tfrac{7}{5}\text{ of a foot.}$$

$$\text{Then }\tfrac{4}{3}\text{ of a fur.} = \frac{4\times40\times5\frac{1}{2}\times{}'\times\frac{5}{7}}{3} = \frac{4400}{7} = \text{fraction of}$$

$\frac{7}{5}$ of a foot. *Ans.*

EXERCISE 37.

1. Reduce $\frac{7}{11}$ of a day to the fraction of a week.
2. Reduce $\frac{4}{29}$ of a cwt. to the fraction of a quarter.
3. Reduce $\frac{2}{7}$ of $\frac{14}{25}$ of $\frac{5}{4}$ of a yard to the fraction of an ell Flemish.
4. Reduce $\frac{3}{4}$ of $\frac{5}{7}$ of $\frac{14}{19}$ of a mile to the fraction of a perch.
5. Reduce $\frac{4}{5}$ of $\frac{7}{8}$ of $3\frac{1}{2}$ inches to the fraction of a linear yard.
6. Reduce $\frac{2}{7}$ of $\frac{3}{8}$ of $\frac{6\frac{1}{2}}{4\frac{3}{4}}$ of 6 oz. to the fraction of $\frac{1}{2}$ of $\frac{3}{7}$ of $\frac{3}{4}$ of a scruple.
7 Reduce $\frac{7}{10}$ of $\frac{4}{11}$ of $\frac{1}{6}$ of $\frac{2\frac{1}{2}}{\frac{1}{7}}$ of a pint to the fraction of $\frac{3}{4}$ of $\frac{2}{5}$ of $\frac{4\frac{3}{4}}{7\frac{1}{2}}$ of a bushel.
8. Reduce $\frac{2}{7}$ of $\frac{4}{11}$ of $6\frac{3}{4}$ shillings to the fraction of one pound.
9. Reduce $\frac{5}{11}$ of $4\frac{3}{4}$ hours to the fraction of a week.
10. Reduce $\frac{2}{4}$ of a lb. to the fraction of $\frac{1}{7}$ of $\frac{3}{8}$ of $\frac{4\frac{1}{4}}{6\frac{7}{8}}$ of $\frac{9\frac{1}{2}}{\frac{3}{11}}$ of a dwt.
11. Reduce $\frac{6}{7}$ of $4\frac{3}{4}$ of $\frac{9\frac{1}{4}}{16\frac{1}{2}}$ of $\frac{33}{37}$ of an acre to the fraction of $\frac{3}{7}$ of a square yard.
12. Reduce $\frac{4\frac{1}{2}}{7}$ of $\frac{6}{3\frac{3}{4}}$ of $\frac{\frac{1}{7}}{\frac{1}{9}}$ of $\frac{7}{9}$ of a farthing to the fraction of a pound.

---

27. To reduce one denominate number to the fraction of another:—

RULE.

Reduce both quantities to the lowest denomination contained in either.

Then place that quantity which is to be the fraction of the other as numerator, and the remaining quantity as denominator.

Example 1.—Reduce 4 lbs. 2 oz. to the fraction of 9 lbs. 7 oz. 11 dwt.

OPERATION.

4 lbs. 2 oz. = 1000 dwt.
9 lbs. 7 oz. 11 dwt. = 2311 dwt.

Therefore 4 lbs. 2 oz. is $\frac{1000}{2311}$ of 9 lbs. 7 oz. 11 dwt.

Example 2.—Reduce 16s. 4¾d. to the fraction of £91 9s. 11d.

OPERATION.

16s. 4¾d. = 787 farthings.
£91 9s. 11d. = 87836 farthings.

Therefore the answer is $\frac{787}{87836}$.

Exercise 38.

1. What fraction is 2 hours 17 minutes of 1 week 17 hours?
2. What fraction is 19 lbs. 7 oz. 21 grs. of 11 lbs. 7 oz. 9 dwt.?
3. What fraction is 6 per. 16 yds. 2 ft. 11 in. of 7 roods 14 perches?
4. What fraction is 3 qrs. 1 na. 1 in. of 3 Eng. e. 1 qr. 2 na.?
5. Reduce 27 weeks 2 days 4 hours 7 min. to the fraction of a year.
6. Reduce 2 qts. 1 pt. to the fraction of 7 bush. 1 pk.
7. Reduce 1 lb. 1 oz. to the fraction of 3 cwt. 3 qrs. 17 lbs.
8. Reduce £176 18s. 7½d. to the fraction of £217 19s. 11d.
9. What fraction is 17 farthings of 6s. 11¾d.?
10. Reduce 27 square yards to the fraction of an acre.
11. What fraction is 7 drs. 1 scr. 17 grs. of 7 lbs. 4 oz. 7 drs.?
12. Reduce ½ of ⅓ of ¾ of £7 8s. 3¼d. to the fraction of ⅝ of ⅞ of $\frac{17\frac{1}{2}}{6}$ of £6 7s. 8½d.

13. Reduce $\frac{2}{7}$ of $\frac{3}{8}$ of $\frac{8}{9}$ of $\frac{21}{20}$ of 1 qr. 17 lbs. to the fraction of $\frac{1}{2}$ of $\frac{2}{5}$ of $\frac{10}{13}$ of 6 cwt. 1 qr.

14. Reduce $\frac{3}{11}$ of $\frac{4}{9}$ of $\frac{16\frac{1}{2}}{20}$ of 6 roods 17 per. to the fraction of $\frac{7}{11}$ of $\frac{8}{3}$ of $\frac{33}{56}$ of $\frac{2}{7}$ of 9 acres 11 yds.

15. Reduce $\frac{1}{2}$ of $7\frac{1}{2}$ of $8\frac{3}{4}$ of $\frac{1}{5}$ of $\frac{16}{25}$ of 17 cord-feet to the fraction of $\frac{9}{11}$ of $5\frac{1}{2}$ of $\frac{1}{3}$ of 3 cords 56 cubic ft.

---

28. To find the value of a denominate fraction in terms of lower denominations:—

RULE.

Consider the numerator as so many units of the given denomination, and divide by the denominator.

EXAMPLE 1.—What is the value of $\frac{11}{13}$ of a mile?

OPERATION.

11 miles ÷ 13 (Art. 22, Sect. II.) = 6 fur. 30 per. 4 yds. 0 ft. $8\frac{4}{13}$ in. *Ans.*

EXAMPLE 2.—What is the value of $\frac{17}{29}$ of a cwt.?

OPERATION.

17 cwt. ÷ 29 = 2 qrs. 8 lbs. 9 oz. $14\frac{26}{29}$ drs. *Ans.*

EXERCISE 39.

Find the value of the following fractions:

1. $\frac{1}{7}$ of a week.
2. $\frac{1}{2}$ of $\frac{3}{4}$ of a bushel.
3. $\frac{2}{5}$ of $\frac{4}{7}$ of $\frac{9\frac{1}{2}}{3\frac{1}{4}}$ of a hhd.
4. $\frac{1}{11}$ of $8\frac{1}{4}$ lbs. Troy.
5. $\frac{2}{7}$ of $\frac{3}{11}$ of $\frac{8\frac{1}{2}}{17}$ of an acre.
6. $\frac{1}{4}$ of $\frac{3}{11}$ of $6\frac{3}{5}$ of $\frac{4}{17}$ of a French ell.
7. $\frac{3}{4}$ of $\frac{3}{5}$ of a £.
8. $7\frac{1}{2}$ of $3\frac{5}{8}$ of $\frac{9\frac{1}{2}}{7\frac{1}{4}}$ of an acre.
9. $7\frac{4}{7}$ of $9\frac{1}{2}$ of $\frac{7}{38}$ of a mile.
10. $\frac{2}{5}$ of $\frac{3}{7}$ of $\frac{4}{9}$ of 35 cwt.
11. $\frac{2}{11}$ of $\frac{8}{7}$ of a lb. Apoth.
12. $\frac{3\frac{1}{2}}{7\frac{1}{5}}$ of $\frac{36}{35}$ of $\frac{2}{11}$ of $\frac{4}{7}$ of a £.

ADDITION OF FRACTIONS.

RULE.

29. *Reduce the fractions to a common denominator, add the numerators together for a new numerator, and beneath their sum write the common denominator.*

Reduce the resulting fraction, if it be an improper fraction, to a mixed number.

NOTE.—If any mixed numbers occur among the addends, add the fractions separately, and to their sum add the sum of the integral portions.

EXAMPLE 1.—Add together $\frac{1}{2}$, $\frac{3}{5}$, $\frac{3}{8}$, $\frac{7}{10}$, and $\frac{4}{15}$.

OPERATION.

By Art. 23, these fractions, reduced to a common denominator, become

$$\frac{60}{120}+\frac{72}{120}+\frac{45}{120}+\frac{84}{120}+\frac{32}{120}=\frac{60+72+45+84+32}{120}=$$

$$\frac{293}{120}=2\tfrac{53}{120}. \textit{ Ans.}$$

EXAMPLE 2.—What is the sum of $6\frac{3}{7}+19\frac{4}{11}+9\frac{3}{8}+17\frac{6}{7}+23\frac{13}{14}$?

OPERATION.

$$6\tfrac{3}{7}+19\tfrac{4}{11}+9\tfrac{3}{8}+17\tfrac{6}{7}+23\tfrac{13}{14}$$

$$=6+19+9+17+23+\tfrac{3}{7}+\tfrac{4}{11}+\tfrac{3}{8}+\tfrac{6}{7}+\tfrac{13}{14}$$

$$6+19+9+17+23=74.$$

$$\tfrac{3}{7}+\tfrac{4}{11}+\tfrac{3}{8}+\tfrac{6}{7}+\tfrac{13}{14}=\tfrac{264}{616}+\tfrac{224}{616}+\tfrac{231}{616}+\tfrac{528}{616}+\tfrac{572}{616}$$

$$=\frac{264+224+231+528+572}{616}=\tfrac{1819}{616}=2\tfrac{587}{616}.$$

Then $74+2\frac{587}{616}=76\frac{587}{616}$. *Ans.*

EXERCISE 40.

Find the value of :—

1. $\frac{1}{2} + \frac{3}{7} + \frac{4}{11} + \frac{5}{6} + \frac{7}{9}$.
2. $\frac{2}{3} + \frac{2}{5} + \frac{2}{7} + \frac{2}{9} + \frac{2}{11} + \frac{2}{15}$.
3. $\frac{1}{4} + \frac{3}{5} + \frac{5}{6} + \frac{2}{3} + \frac{7}{10}$.
4. $\frac{2}{5} + 1\frac{3}{4} + 2\frac{1}{7} + 9\frac{1}{2}$.
5. $6\frac{1}{4} + 11\frac{2}{3} + 196\frac{1}{5} + 29\frac{6}{7}$.
6. $8\frac{1}{4} + 11\frac{1}{12} + \frac{6}{7} + \frac{4}{5} + 16\frac{5}{9}$.
7. $196\frac{3}{7} + 16\frac{17}{19} + 20\frac{4}{11}$.
8. $200\frac{1}{2} + 763\frac{1}{5} + 916\frac{3}{7}$.
9. $17\frac{16}{17} + 19\frac{18}{19} + 20\frac{19}{20} + 21\frac{20}{21}$.
10. $6\frac{1}{3} + 8\frac{1}{4} + 11\frac{3}{4} + 9\frac{1}{5} + 16\frac{2}{7}$.
11. $\frac{1}{2}$ of $\frac{3}{7} + \frac{1}{8}$ of $\frac{4}{9} + \frac{6}{7}$ of $6\frac{2}{7}$.
12. $\frac{2}{5}$ of $\frac{1}{2}$ of $\frac{7}{8} + 9\frac{1}{8} + 6\frac{3}{11} + \frac{2}{7}$ of $\frac{4}{9}$ of $\frac{63}{80}$.
13. $7\frac{3}{4} + 9\frac{1}{2} + 13\frac{3}{7} + 20\frac{1}{2} + \frac{1}{2}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{9}{11}$.
14. $6\frac{3}{7} + \frac{1}{2}$ of $\frac{4}{5}$ of $10 + \frac{2}{11}$ of $\frac{6}{11}$ of $242 + 16\frac{3}{11}$.
15. $111\frac{11}{12} + 22\frac{2}{3} + 3\frac{3}{4} + \frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$.
16. $67\frac{7}{8} + 89\frac{9}{10} + 90\frac{10}{11} + 101\frac{101}{107}$.
17. $\dfrac{8\frac{3}{4}}{6\frac{1}{2}} + \dfrac{9\frac{2}{7}}{8\frac{3}{11}} + \dfrac{16\frac{1}{5}}{9\frac{3}{7}}$.
18. $\frac{1}{2}$ of $\frac{2}{7}$ of $4 + 6\frac{1}{2}$ of $9\frac{3}{7} + 18\frac{1}{4} + \frac{20}{21}$.
19. $2\frac{3}{4} + \dfrac{16\frac{1}{4}}{13} + \dfrac{7\frac{1}{2}}{8\frac{3}{7}} + \frac{5}{12}$ of $\frac{7}{11}$ of $\frac{132}{175}$.
20. $\dfrac{4\frac{1}{4}}{\frac{1}{7}}$ of $\dfrac{6\frac{1}{2}}{11} + 9\frac{5}{6} + 11\frac{1}{2} + 16\frac{3}{7} + \dfrac{7\frac{3}{7}}{4\frac{1}{8}}$.

SUBTRACTION OF FRACTIONS.

RULE.

30. *Reduce both of the fractions to a common denominator.*

Subtract the numerator of the subtrahend from the numerator of the minuend, and beneath the difference write the common denominator.

NOTE.—In the case of mixed numbers, it frequently happens that the fractional part of the subtrahend is greater than the fractional part of the minuend. When this occurs, instead of reducing both quantities to improper fractions and then applying the rule, it is much better to borrow one from the integral part of the minuend, and, considering it as a fraction having the common denominator, add it to the fractional part of the minuend.

EXAMPLE 1.—From $\frac{16}{17}$ take $\frac{9}{13}$.

OPERATION.

$\frac{16}{17} - \frac{9}{13} = \frac{208}{221} - \frac{153}{221} = \frac{208-153}{221} = \frac{55}{221}$. *Ans.*

EXAMPLE 2.—From $11\frac{3}{7}$ take $9\frac{11}{12}$.

OPERATION.

$11\frac{3}{7} - 9\frac{11}{12} = 11\frac{36}{84} - 9\frac{77}{84} = 10 + 1\frac{36}{84} - 9\frac{77}{84} = 10\frac{120}{84} - 9\frac{77}{84} = 1\frac{43}{84}$. *Ans.*

EXAMPLE 3.—From $\frac{2}{7}$ of $\frac{16}{19}$ of $1\frac{1}{6}$ take $\frac{17}{190}$.

OPERATION.

$\frac{2}{7}$ of $\frac{16}{19}$ of $1\frac{1}{6} - \frac{17}{190} = \frac{16}{57} - \frac{17}{190} = \frac{160}{570} - \frac{51}{570} = \frac{109}{570}$. *Ans.*

EXERCISE 41.

Find the value of:

1. $\frac{7}{11} - \frac{4}{17}$.
2. $\frac{5}{6}$ of $\frac{4}{5} - \frac{2}{3}$ of $\frac{1}{5}$.
3. $\frac{3}{11}$ of $6\frac{5}{8} - \frac{1}{2}$ of $2\frac{3}{7}$.
4. $169\frac{3}{7} - 23\frac{17}{18}$.
5. $229\frac{1}{16} - 67\frac{13}{24}$.
6. $1116\frac{1}{4} - 229\frac{14}{19}$.
7. $11\frac{11}{13} - 1\frac{13}{14}$.
8. $196\frac{1}{2} - \dfrac{14\frac{1}{2}}{16\frac{1}{7}}$ of $\dfrac{11\frac{2}{7}}{9\frac{1}{5}}$.
9. $\frac{2}{7}$ of $\frac{4}{5}$ of $18\frac{3}{11}$ of $2\frac{39}{57} - \frac{7}{8}$ of $\frac{1}{4}$ of $\frac{44}{7}$ of $\frac{31}{6}$.
10. $\frac{1}{2} + \frac{2}{3} + \frac{7}{10} - \frac{7}{11} + \frac{4}{7} - \frac{6}{17} - \frac{9}{10}$.
11. $16\frac{1}{2} + 4\frac{3}{8} + 16\frac{3}{8} + 20\frac{1}{6} - 17\frac{4}{11}$.
12. $4\frac{1}{8}$ of $\dfrac{16\frac{1}{2}}{11\frac{3}{8}} - \dfrac{2}{7}$ of $\dfrac{16\frac{1}{4}}{17\frac{3}{4}}$.
13. $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{4}{7}$ of $16\frac{4}{7} - 11\frac{1}{4} + 7\frac{6}{7} + 11\frac{1}{10} - \frac{3}{8}$.
14. $96\frac{3}{8} - \frac{1}{2}$ of $\frac{4}{7}$ of $\frac{1}{3}$ of $63 + \frac{3}{11} + 18\frac{1}{2} - 17\frac{13}{18}$.

15. $\frac{11}{12}$ of $\frac{8\frac{3}{4}}{7\frac{1}{8}} - \frac{9}{11}$ of $\frac{6\frac{1}{2}}{8\frac{3}{7}}$.

16. $4\frac{3}{4}$ of $6\frac{1}{2}$ of $7\frac{2}{9} - \frac{2}{3}$ of $8\frac{6}{7}$ of 11.

MULTIPLICATION OF FRACTIONS.

RULE.

31. *Reduce all mixed numbers to improper fractions, and complex and compound fractions to simple ones. Cancel the factors that are common to a numerator and a denominator of the resulting fractions.*

Multiply all the reduced numerators together for a new numerator, and all the reduced denominators together for a new denominator.

Reduce the result, if necessary, to a mixed number.

Example 1.—Multiply $\frac{3}{5}$ by $\frac{15}{17}$.

$$\frac{3}{5} \times \frac{15}{17} = \frac{3}{1} \times \frac{3}{17} = \frac{9}{17}. \text{ Ans.}$$

Here we cancel the first denominator and reduce the second numerator to 3.

Example 2.—Multiply together $\frac{7}{11}$, $\frac{4}{5}$, $3\frac{1}{2}$, and $\frac{55}{98}$.

STATEMENT. CANCELLED.

$$\frac{7}{11} \times \frac{4}{5} \times \frac{7}{2} \times \frac{55}{98} = \frac{\cancel{7}}{\cancel{11}} \times \frac{\overset{\cancel{2}}{\cancel{4}}}{\cancel{5}} \times \frac{\cancel{7}}{\cancel{2}} \times \frac{\overset{\cancel{5}}{\cancel{55}}}{\underset{\underset{\cancel{7}}{\cancel{49}}}{\cancel{98}}} = \frac{1}{1} = 1. \text{ Ans.}$$

Example 3.—Multiply together $\frac{4}{9}$, $\frac{3}{11}$, $6\frac{2}{7}$, $9\frac{3}{5}$, $2\frac{1}{2}$, and 63.

STATEMENT.

$$\frac{4}{9} \times \frac{3}{11} \times \frac{44}{7} \times \frac{48}{5} \times \frac{5}{2} \times \frac{63}{1}.$$

CANCELLED.

$$= \frac{\overset{2}{\cancel{4}}}{\cancel{9}} \times \frac{3}{\cancel{11}} \times \frac{\overset{4}{\cancel{44}}}{\cancel{7}} \times \frac{48}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{2}} \times \frac{\overset{\cancel{7}}{\cancel{63}}}{1} = \frac{2 \times 3 \times 4 \times 48}{1} = 1152. \text{ Ans.}$$

Example 4.—Multiply together $\frac{1}{179}$, $18\frac{7}{11}$, $9\frac{3}{5}$, $\frac{1}{2}$ of $\frac{3}{4}$ of 7 and $\frac{3}{5}$ of $\frac{11}{14}$ of 25.

STATEMENT.

$$\frac{1}{179} \times \frac{205}{11} \times \frac{48}{5} \times \frac{21}{8} \times \frac{165}{14}.$$

CANCELLED.

$$= \frac{1}{179} \times \frac{205}{\cancel{11}} \times \frac{\overset{3}{\overset{\cancel{6}}{\cancel{48}}}}{\cancel{5}} \times \frac{\overset{3}{\cancel{21}}}{\cancel{8}} \times \frac{\overset{3}{\overset{\cancel{33}}{\cancel{165}}}}{\underset{\cancel{2}}{\cancel{14}}} = \frac{205 \times 3 \times 3 \times 3}{179}$$

$$= \frac{5535}{179} = 30\frac{165}{179}.$$

Exercise 42.

Find the value of—

1. $\frac{1}{2} \times \frac{3}{7} \times \frac{9}{11} \times \frac{4}{81} \times 9.$
2. $\frac{2}{7} \times \frac{4}{11}.$
3. $\frac{6}{11} \times 2\frac{1}{4} \times \frac{4}{7} \times 3\frac{1}{2}.$
4. $6\frac{2}{7} \times 4\frac{3}{11} \times 77 \times 4\frac{1}{4}.$
5. $3 \times 7\frac{1}{2} \times \frac{11}{15} \times 3\frac{8}{11}.$
6. $9\frac{3}{5} \times \frac{4}{11} \times 2 \times \frac{3}{17} \times \frac{24}{25}.$
7. $8\frac{3}{4} \times 9\frac{1}{3} \times 10\frac{1}{5} \times \dfrac{1}{9\frac{1}{2}}.$
8. $\frac{6}{7}$ of $\frac{1}{3}$ of $(\frac{1}{2} + \frac{3}{8}) \times \frac{9}{11}$ of $\frac{7}{15}.$
9. $27\frac{3}{5} \times 98\frac{3}{11}.$
10. $16\frac{1}{2} \times 8\frac{1}{4} \times \frac{17}{32} \times \dfrac{10}{10\frac{1}{2}}.$
11. $(11\frac{5}{6} + 6\frac{1}{3}) \times (9\frac{3}{5} - 7\frac{1}{7}).$
12. $\dfrac{4\frac{2}{3}}{7\frac{1}{2}} \times \dfrac{6\frac{1}{3}}{\frac{1}{2}} \times \frac{1}{3}$ of $3\frac{3}{7}$ of $9\frac{3}{8}.$
13. $6\frac{3}{7}$ of $8\frac{2}{3} \times 9\frac{1}{3}$ of $\dfrac{16}{8\frac{2}{3}}.$

14. $\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{6}{7} \times \frac{8}{9} \times \frac{10}{11} \times 693$.

15. $(4\frac{3}{4} - 2\frac{1}{9}) \times 2\frac{1}{8}$ of $4\frac{1}{2}$ of $(7\frac{1}{4} - 6\frac{1}{5})$.

16. $6\frac{3}{11}$ of $\dfrac{4\frac{1}{4}}{7\frac{3}{8}}$ of $\dfrac{1}{9\frac{1}{4}} \times \dfrac{8\frac{3}{5}}{9} \times 11\frac{1}{7}$.

---

17. Multiply $\frac{2}{7}$ of $\frac{3}{11}$ of $\dfrac{37\frac{1}{2}}{9\frac{1}{6}}$ by $\dfrac{4\frac{1}{2}}{7} \times \frac{3}{8} \times 1\frac{5}{9}$.

18. What is the product of $\frac{2}{5} \times \frac{3}{11} \times \frac{4}{7} \times \frac{9}{13} \times 6\frac{1}{2} \times \dfrac{1}{4\frac{1}{2}} \times \dfrac{1}{\frac{3}{14}}$?

19. Find the value of $(6\frac{3}{5} + 4\frac{1}{2} + 9\frac{3}{4}) \times (6\frac{2}{11} + 3\frac{1}{7}) \times (3\frac{1}{5} - 2\frac{1}{9})$.

20. What is the product of $(8\frac{3}{11} - 2\frac{1}{7} + 3\frac{1}{10} - 7\frac{2}{5}) \times (6\frac{3}{11} - 2\frac{7}{9} + 2 + \frac{1}{3}) \times (\frac{1}{2}$ of $11\frac{1}{2} + \frac{6}{7})$?

DIVISION OF FRACTIONS.

RULE.

32. *Reduce compound and complex fractions to simple ones, and whole and mixed numbers to improper fractions.*

Invert the terms of the divisor, and proceed as in multiplication.

Example 1.—Divide $\frac{3}{19}$ by $\frac{4}{11}$.

OPERATION.

$$\tfrac{3}{19} \div \tfrac{4}{11} = \tfrac{3}{19} \times \tfrac{11}{4} = \tfrac{33}{76}.\ \textit{Ans.}$$

Example 2.—Divide $\frac{3}{4}$ of $\frac{7}{11}$ by $\frac{2}{11}$ of $8\frac{3}{4}$.

OPERATION.

$$\tfrac{3}{4} \text{ of } \tfrac{7}{11} \div \tfrac{2}{11} \text{ of } \tfrac{35}{4} = \tfrac{21}{44} \div \tfrac{35}{22} = \tfrac{21}{44} \times \tfrac{22}{35} = \tfrac{3}{10}.\ \textit{Ans.}$$

Example 3.—Divide $8\frac{4}{7}$ by $3\frac{3}{11}$.

OPERATION.

$$8\tfrac{4}{7} \div 3\tfrac{3}{11} = \tfrac{60}{7} \div \tfrac{36}{11} = \tfrac{60}{7} \times \tfrac{11}{36} = 2\tfrac{13}{21}.\ \textit{Ans.}$$

Example 4.—Divide $\frac{3}{17}$ of $\frac{4}{11}$ of $\dfrac{8\frac{3}{4}}{\frac{3}{11}} \times 3\frac{1}{7}$ by $\frac{4}{17}$ of $\dfrac{9\frac{3}{7}}{8\frac{3}{4}} \times 4\frac{3}{8}$.

STATEMENT.

$$\tfrac{3}{17} \times \tfrac{4}{11} \times \tfrac{385}{12} \times \tfrac{22}{7} \div \tfrac{4}{17} \times \tfrac{264}{245} \times \tfrac{35}{8}.$$

TERMS OF DIVISOR INVERTED.

$$= \tfrac{3}{17} \times \tfrac{4}{11} \times \tfrac{385}{12} \times \tfrac{22}{7} \times \tfrac{17}{4} \times \tfrac{245}{264} \times \tfrac{8}{35}.$$

CANCELLED.

$$= \frac{\cancel{3}}{\cancel{17}} \times \frac{\cancel{4}}{\cancel{11}} \times \frac{\overset{11}{\cancel{385}}}{\underset{4}{\cancel{12}}} \times \frac{\overset{2}{\cancel{22}}}{\cancel{7}} \times \frac{\cancel{17}}{\underset{2}{\cancel{4}}} \times \frac{\overset{35}{\cancel{245}}}{\underset{\underset{3}{\cancel{24}}}{\cancel{264}}} \times \frac{\cancel{8}}{\cancel{35}} = \frac{35}{6} = 5\tfrac{5}{6}.\ Ans.$$

Exercise 43.

Find the value of—

1. $\frac{2}{7} \div \frac{4}{11}$.
2. $\frac{2}{7}$ of $\frac{3}{5} \div 4\frac{3}{4}$.
3. $7\frac{5}{8} \div \frac{1}{2}$ of $\frac{3}{4}$ of $6\frac{1}{2}$.
4. $\frac{6}{7}$ of $\frac{4}{5} \div \frac{3}{5}$ of $\frac{7}{9}$.
5. $3\frac{3}{5}$ of $8\frac{1}{2} \div 6\frac{2}{5}$ of $5\frac{3}{7}$.
6. $\frac{4}{5}$ of $3\frac{7}{8}$ of $9\frac{1}{2} \div \dfrac{6\frac{3}{4}}{9\frac{2}{7}}$.
7. $\frac{2}{5}$ of $8\frac{3}{4}$ of $6\frac{2}{7} \div 4\frac{3}{4}$ of $2\frac{1}{7}$.
8. $(\frac{1}{2}+\frac{3}{4}+\frac{4}{5}-\frac{6}{7}) \div \frac{2}{5}$ of $\frac{1}{3}$.
9. $\dfrac{4\frac{2}{5}}{8\frac{1}{7}} \div \dfrac{8\frac{1}{5}}{17\frac{3}{5}}$.
10. $8\frac{2}{7}$ of $4\frac{3}{5}$ of $6\frac{1}{4} \div 3\frac{1}{7}$ of $\dfrac{6\frac{3}{4}}{5\frac{1}{5}}$.
11. $4\frac{1}{4} \div 6\frac{1}{2}$ of $\frac{3}{11}$ of $4\frac{1}{4}$.
12. $9\frac{1}{4}$ of $\dfrac{8\frac{1}{2}}{\frac{7}{8}} \div 6\frac{3}{5}$ of $\dfrac{4\frac{2}{7}}{8\frac{1}{4}}$.
13. $(\frac{3}{5}$ of $\frac{7}{8}$ of $8\frac{3}{4}) \div (\frac{3}{7}$ of $4\frac{3}{5} - \frac{13}{20})$.
14. $8\frac{3}{4}$ of $6\frac{1}{2}$ of $4 \times \frac{3}{8}$ of $\frac{4}{5}$ of $\frac{1}{13} \times \frac{1}{7} \div 6\frac{3}{4}$ of $\dfrac{8\frac{1}{2}}{7\frac{1}{8}}$.
15. $9\frac{1}{3}$ of $8\frac{3}{4}$ of $6\frac{2}{7} \div \dfrac{8\frac{2}{7}}{6\frac{1}{5}}$ of $\dfrac{6\frac{2}{7}}{4\frac{1}{14}}$.

---

33. To multiply an integral denominate number by a fraction:—

RULE.

Multiply the denominate number by the numerator of the fraction, and divide the result by the denominator.

Example.—How much is $\frac{3}{11}$ of $\frac{1}{2}$ of $\frac{3}{7}$ of $4\frac{2}{3}$ of 4 days 21 hours 7 minutes?

OPERATION.

$\frac{3}{11}$ of $\frac{1}{2}$ of $\frac{3}{7}$ of $4\frac{2}{3}$ of 4 d. 21 h. 7 m. $=\frac{3}{11}$ of 4 d. 21 h. 7 m.

$$=\frac{\text{4 d. 21 h. 7 m.} \times 3}{11}=\text{1 d. 7 h. 56 m. } 27\tfrac{3}{11}\text{ sec. } \textit{Ans.}$$

34. To divide an integral denominate number by a fraction:—

RULE.

Multiply the denominate number by the denominator of the fraction and divide the result by the numerator.

Example.—Divide 7 lbs. 4 oz. 7 dwt. by $\frac{1}{3}$ of $4\frac{3}{4}$ of $6\frac{1}{5}$ of $\dfrac{1}{15\frac{1}{2}}$.

OPERATION.

$$\text{7 lb. 4 oz. 7 dwt.} \div \tfrac{1}{3}\text{ of }4\tfrac{3}{4}\text{ of }6\tfrac{1}{5}\text{ of }\frac{1}{15\frac{1}{2}}=\text{7 lbs. 4 oz. 7 dwt.}\div\tfrac{19}{30}=\frac{\text{7 lbs. 4 oz. 7 dwt.}\times 19}{30}=\text{4 lbs. 7 oz. 19 dwt. } 2\tfrac{2}{5}\text{ grs. } \textit{Ans.}$$

Exercise 44.

Find the value of—

1. $\frac{7}{9}$ of $\frac{8}{5}$ of £1 16s. $8\frac{1}{2}$d.
2. $3\frac{1}{4}$ of $8\frac{1}{2}$ of $\frac{3}{13}$ of $\frac{1}{17}$ of 4 bush. 1 pk. 1 pt.
3. $\frac{1}{5}$ of $\frac{8}{9}$ of $6\frac{3}{4}$ of $5\frac{2}{7}$ of $\frac{11}{76}$ of 6 lbs. 4 oz. Avoir.
4. $\frac{11}{16}$ of $\frac{3}{7}$ of $\frac{8}{33}$ of $6\frac{1}{4}$ acres.
5. $\frac{1}{2}$ of $\frac{4}{7}$ of 11 cwt. 1 qr. 11 lbs.
6. $\frac{6}{7}$ of $\frac{4}{5}$ of $\frac{11}{12}$ of 3 acres 1 rood 27 per.
7. $6\frac{3}{4}$ of $\frac{11}{18}$ of $\frac{3}{22}$ of £6 11s. $4\frac{3}{4}$d.
8. $6\frac{1}{4}$ of $11\frac{1}{2}$ of $2\frac{1}{4}$ of $\frac{9}{23}$ of 7 miles 4 fur. 17 per.
9. $\frac{1}{2}$ of $\frac{7}{11}$ of 3 lbs. 5 oz. Avoir. + $6\frac{3}{4}$ of $\frac{1}{27}$ of 6 lbs. 11 oz.
10. $2\frac{3}{4}$ of $\frac{7}{5}$ of 6 lbs. 11 oz. 4 drs. 1 scr. 16 grs.

11. $4\frac{3}{4}$ of $5\frac{1}{2}$ of $\frac{7}{19}$ of $\frac{6}{11}$ of 4 yds. 3 qrs. 2 na.

12. $7\frac{1}{4}$ of $\dfrac{1}{14\frac{1}{2}}$ of $6\frac{3}{11}$ of 2 qrs. 17 lbs. 4 oz.

13. $\frac{7}{18}$ of $6\frac{1}{4}$ of $\frac{16}{25}$ of 21 bush. 3 pks.

14. $\frac{6}{11}$ of $3\frac{1}{4}$ of $\frac{7}{12}$ of $\frac{11}{13}$ of $\dfrac{6\frac{1}{4}}{7\frac{1}{8}}$ of 7 wk. 4 d. 5 h.

15. 21 lbs. 11 oz. 7 dwt. $\div \frac{6}{7}$ of $\frac{4}{5}$ of $17\frac{1}{2}$.

16. 4 acres 6 per. 5 yds. $\div 5\frac{1}{2} \times \frac{3}{7} \times \frac{16}{33}$.

17. £169 4s. $11\frac{1}{2}$d. $\div 3\frac{1}{4}$ of $6\frac{1}{2}$ of $\frac{119}{169}$.

18. 11 cwt. 2 qrs. 17 lbs. $\times 6\frac{3}{4}$ of $4\frac{1}{7}$ of $\frac{1}{127}$.

DECIMALS.

35. A Decimal Fraction is a fraction which has 10, 100, 1000, 10000, &c., for its denominator.

Thus $\frac{7}{10}$, $\frac{11}{100}$, $\frac{27}{1000}$, $\frac{643}{10000}$, $\frac{9}{100000}$, &c., are decimal fractions.

36. In writing decimals it is customary to omit the denominator, and to place a dot, called the decimal point, before the numerator.

Thus $\frac{7}{10}$ is written ·7 ; $\frac{11}{100}$ is written ·11 ; $\frac{27}{1000}$ is written ·027 ; $\frac{643}{10000}$ is written ·0643 ; $\frac{9}{100000}$ is written ·00009, &c.

37. The *orders* to the right of the decimal point are named as follows: *tenths*, *hundredths*, *thousandths*, *tenths of thousandths*, *hundredths of thousandths*, *millionths*, *tenths of millionths*, &c.

38. The *periods* to the right of the decimal point are named as follows: *thousandths*, *millionths*, *billionths*, *trillionths*, *quadrillionths*, &c.

39. In order to read any decimal, we proceed according to the following

RULE.

Point off into periods of three figures each, commencing at the decimal point, and thus ascertain the name of *order of the extreme right-hand figure. Then find by simble numeration, from right to left, how many of that order are indicated.*

Example 1.—Read ·0000006174.

Here pointing off the given expression, it becomes ·000,000,617,4, from which we learn that the denomination is that of *tenths of billionths*, and the expression represents six thousand one hundred and seventy-four tenths of billionths.

Example 2.—Read 417·00000007146174.

Here pointing off, we get 417·000,000,071,461,74, i. e. the right-hand digit is that of *hundredths of trillionths*, and the decimal part is 7146174 hundredths of trillionths. Hence the whole expression is read—Four hundred and seventeen, and seven millions one hundred and forty-six thousand one hundred and seventy-four *hundredths of trillionths.*

Exercise 45.

Read the following expressions:

1. ·27; ·043; ·007; ·6914; ·008196; ·00071423.
2. 6·7; 93·42; 147·1394; 217·0000009.
3. 71·00089; 167·193; 91·0008674.
4. 5674378·000914786; 71300400·000000600407.

---

40. To write any decimal, we proceed according to the following

RULE.

Ascertain how many places to the right of the decimal point the given denomination comes.

Then if the given digits do not occupy all of these places, insert between them and the decimal point the requisite number of ciphers.

Example 1.—Write down seventy-eight tenths of thousandths.

Here, since tenths of thousandths occupy the *fourth* place to the right of the decimal point, and since the given digits, 78, are only *two* in number, we must insert the two ciphers between the decimal point and the 7, and the number is then written

·0078.

EXAMPLE 2.—Write down seven hundred and four thousand and ninety-one hundredths of trillionths.

Here, since the periods to the right of the decimal point are thousandths, millionths, billionths, trillionths, &c., we find that trillionths occupy the 12th place, and consequently *hundredths of trillionths* the 14th place to the right of the decimal point. Now the given number, 704091, only contains *six* digits, and consequently we must insert between the decimal point and the 7 the difference between 14 and 6, i. e. 8 ciphers, when the expression becomes—

·00000000704091.

EXERCISE 46.

Write down the following numbers:

1. Nine hundredths; seventy-six tenths of thousandths; four hundred and forty-seven millionths.
2. Seven hundred thousand and sixteen hundredths of billionths.
3. Five millions twenty-nine thousand and eleven tenths of trillionths.
4. Eighty-seven thousand four hundred and three tenths of millionths.
5. Seven hundred and nine thousandths.
6. Four hundred and ninety-six thousand seven hundred and nineteen, *and* eleven thousand and four tenths of millionths.
7. Seven millions six hundred and ninety-one thousand and six, *and* fourteen millions seven hundred thousand nine hundred and thirty trillionths.

8. Express as a decimal 17 tenths + 17 thousandths + 17 hundredths of thousandths.
9. Express decimally 749 units + 2 tenths + 40 tenths of millionths.
10. Express as a decimal 7297 hundredths + 704 millionths.

41. To multiply a decimal by 10, 100, 1000, &c., we remove the decimal point 1, 2, 3, &c. places to the right; and to divide by 10, 100, 1000, &c., we remove it similarly to the left.

Thus, ·006 × 10 = ·06; ·006 × 100 = ·6; ·006 × 10000 = 60, &c.
·867 ÷ 10 = ·0867; ·867 ÷ 100000 = ·00000867, &c.

Note.—Affixing ciphers to a decimal, i. e. writing them to the right of it, does not alter its value, thus, ·79, ·790, and ·79000 are all equal, each representing $\frac{79}{100}$; but prefixing ciphers to a decimal, i. e. writing them between the decimal point and the left-hand figure of the decimal, divides the decimal by 10 for each cipher thus prefixed.

ADDITION AND SUBTRACTION OF DECIMALS.

RULE.

42. *Write the numbers to be added or subtracted, so that the decimal points shall be directly under one another, and proceed as in addition and subtraction of whole numbers, being careful to place the decimal point in the answer in the same vertical line with the others.*

Example 1.—Add together 78·647, ·0078, 9·816, 4·278, 967·4278, and ·0091.

OPERATION.

78·647
·0078
9·816
4·278
967·4278
·0091
1060·1857

Here we begin at the right-hand side, as in simple addition, and proceed as follows: 1 and 8 make 9 and 8 make 17, set down 7 and carry 1; 1 and 9 make 10 and 7 make 17, &c.

EXAMPLE 2.—From 71·0047 take 9·0008167.

OPERATION.

```
71·0047
 9·0008167
62·0038833
```

Here we say 7 from 0 we can't, but 7 from 10 and 3 remain; 7 (i. e. 6 and 1 carried) from 0 we can't, but 7 from 10 and 3 remain; 2 from 0 we can't, but 2 from 10 and 8 remain; 9 from 7 we can't, but 9 from 17, &c.

EXERCISE 47.

Find the value of—

1. 18·716 + 967 + 34·71 + ·271 + 698·7149 + 23·0067.
2. 278·714 + 61·9134 + 217·8167 + 23·7146 + 678·906 + 12·98678.
3. 216·714763 + 2·9 + 9867 + 91·0986 + 7·81645 + ·09868.
4. 26·1111 + 11·22222 + 34·546 + 17·19186 + 11·127 + 816·7142.
5. ·9167 + 9·9 + 8·98 + 7·614 + ·0986 + 17 + 19·11 + 963·714.
6. 9·64 + 9111·77 + 967·769 + 463 + 7·0009 + 8·61 + 911·1257.
7. 167·914 — 6·8147.
8. 9161·0098 — 7149·16716.
9. 71·0916714 — 27·1471.
10. 1111·116 — 22·22222.
11. 279·00906 — 117·916.
12. 627·4 — 91·7469.

MULTIPLICATION OF DECIMALS.

RULE.

43. *Multiply the given decimals as though they were whole numbers, and mark off in the product as many decimal places as there are in the multiplier and multiplicand together.*

EXAMPLE 1.—Multiply ·743 by ·067.

OPERATION.

$$\begin{array}{r} \cdot743 \\ \cdot067 \\ \hline 5201 \\ 4458 \\ \hline \cdot049781 \end{array}$$

Here, when we multiply 743 and 67 together, we get as the product 49781, but since the decimal places of the multiplier and multiplicand number *six*, we must have six decimal places in the product, and, in order to make the requisite number, we have to prefix one cipher.

Example 2.—Multiply 5·69 by 30·8.

OPERATION.

569 × 308 = 175252, and, since the decimal places of the multiplier together with those of the multiplicand number *three*, we place the point so as to have three places in the product, and this gives us 175·252 for the answer.

Exercise 48.

Find the value of—

1. 78·417 × 9.
2. 27·1 × 3·4.
3. 21·716 × 2·06.
4. 11·007 × ·00678.
5. 1167·91 × 8·100004.
6. 11·111 × 9·7116.
7. 2·7 × 1·4 × 1·19.
8. 3·42 × ·061 × ·0079.
9. 41·1467 × 6·1 × 2·7.
10. 80·08 × 6·6 × 20·02.
11. 1·012 × ·00719.
12. ·2 × ·7 × ·06 × ·041.

DIVISION OF DECIMALS.

RULE.

44. *Divide as in whole numbers, merely remembering to place the decimal point in the quotient directly after the unit's figure of the dividend has been brought down.*

Note.—When the *divisor* contains decimals, before dividing remove the decimal point to the right of the divisor, and also remove it as many *places* to the right in the dividend.

Example 1.—Divide 716·193 by 614.

OPERATION.

$$\begin{array}{l}
614)716\cdot193(1\cdot16643,\ \&c.\\
\quad 614\\ \hline
\quad 1021\\
\quad\ \ 614\\ \hline
\qquad 4079\\
\qquad 3684\\ \hline
\qquad\ \ 3953\\
\qquad\ \ 3684\\ \hline
\qquad\quad 2690\\
\qquad\quad 2456\\ \hline
\qquad\qquad 2340\\
\qquad\qquad 1842\\ \hline
\qquad\qquad\ \ 498,\ \&c.
\end{array}$$

Here we divide as in whole numbers, but, when we have brought down the unit's figure, which in this example is in the first step of the division, we place the decimal point in the quotient. After bringing down the last figure in the decimal, we may continue the division by bringing down ciphers. (See Art. 39.)

Example 2.—Divide 7·43 by ·0079.

OPERATION.

$$7\cdot43 \div \cdot0079 = 74300 \div 79 \doteq 940\cdot5063,\ \&c.$$

Here, since the divisor contains decimals, we remove the decimal point to the right of it, and also as many places, i. e. *four* places, to the right in the dividend; this gives us 74300 ÷ 79.

45. The following will illustrate the mode of thus preparing numbers for division when the divisor contains decimals:

$67\cdot9 \div \cdot9 = 679 \div$ by 9.

$27\cdot09 \div \cdot0047 = 270900 \div 47.$

$27\cdot14678 \div 2\cdot47 = 2714\cdot678 \div 247.$

$114\cdot00672 \div 6\cdot0437 = 1140067\cdot2 \div 60437.$

$278 \div \cdot0147 = 2780000 \div 147.$

$2\cdot614789 \div 27\cdot1434 = 26147\cdot89 \div 271434.$

EXERCISE 49.

Find the value, each to three places of decimals, of :

1. 78·1 ÷ 1·071.
2. 91·142 ÷ 7·8.
3. 61·123 ÷ ·0146.
4. 9·1234 ÷ ·000716.
5. ·0467 ÷ ·01471.
6. 918 ÷ 914·71.
7. 967·104 ÷ 12·046.
8. 91·671 ÷ ·000916.
9. 8·8 ÷ ·0641.
10. 7147·12 ÷ 1127.
11. ·817 ÷ ·9147.
12. 213 ÷ 91·614.

REDUCTION OF DECIMALS.

46. To reduce a vulgar fraction to a decimal :—

RULE.

Divide the numerator by the denominator.

EXAMPLE 1.—Reduce $\frac{7}{8}$ to a decimal.

OPERATION.

$$\begin{array}{r} 8)\underline{7} \\ \cdot 875 \end{array}\quad \textit{Ans.}$$

EXAMPLE 2.—Reduce $\frac{14}{31}$ to a decimal.

OPERATION.

$$14 \div 31 = \cdot 451612903. \quad \textit{Ans.}$$

EXERCISE 50.

Reduce the following fractions to their equivalent decimals.

1. $\frac{2}{7}$ and $\frac{4}{9}$.
2. $\frac{6}{11}$, $\frac{9}{5}$, and $\frac{7}{10}$.
3. $\frac{12}{13}$, $\frac{2}{11}$, and $\frac{19}{20}$.
4. $\frac{167}{240}$, $\frac{11}{19}$, and $\frac{17}{49}$.
5. $\frac{67}{81}$ and $\frac{217}{491}$.
6. $\frac{7}{111}$ and $\frac{29}{187}$.
7. $\frac{614}{1127}$ and $\frac{916}{29}$.
8. $\frac{71}{82}$ and $\frac{167}{271}$.
9. $\frac{61423}{71419}$ and $\frac{167147}{9816}$.
10. $\frac{243}{17}$ and $\frac{123456789}{987654321}$.

47. Sometimes in making these reductions the division never terminates, but a figure or set of figures constantly recurs. Thus in the first part of Example 1, the figures

285714, and in the last part the figure 4, constantly recur. In this case the decimal is called a *repeater* or *circulator*.

48. Decimals which do not terminate, i. e. which consist of the same digit or set of digits constantly repeated, are called Repeating or Circulating Decimals.

49. The digits or set of digits which repeats, is called a *repetend*, *period*, or *circle*.

NOTE.—The terms *period* and *circle* are used only when the repetend contains two or more digits.

50. A Single Repetend is one in which only a single digit repeats.

Thus, ·3333, &c.; ·7777, &c.; ·88888, are single repetends.

51. A Single Repetend is expressed by writing the digit that repeats with a dot over it.

Thus, ·333, &c., is written $\cdot\dot{3}$; ·777, &c., is written $\cdot\dot{7}$.

52. A Circulating Decimal or Compound Repetend is one in which more than one digit repeats.

Thus, ·347347347, &c.; ·202020, &c.; ·123412341234, &c., are circulating decimals or compound repetends.

53. A Circulating Decimal is expressed by writing the recurring period once with a dot over its first and last digits.

Thus, ·347347, &c., is written $\cdot\dot{3}4\dot{7}$; ·2020, &c., $\cdot\dot{2}\dot{0}$; ·12341234, &c., $\cdot\dot{1}23\dot{4}$.

54. A Pure Repetend or Circulating Decimal is one in which the repetend commences *immediately* after the decimal point.

55. A Mixed Repetend or Circulating Decimal is one which contains one or more ciphers or significant figures between the repetend and the decimal point.

Thus $\cdot\dot{3}$, $\cdot\dot{7}$, $\cdot\dot{1}$ are pure repetends.

$\cdot 7891\dot{7}$, $\cdot 0\dot{3}7\dot{8}$, $\cdot 00\dot{2}$ are mixed repetends.

$\cdot\dot{7}\dot{2}$, $\cdot\dot{0}4\dot{3}$, $\cdot\dot{8}137\dot{0}$ are pure circulating decimals.

$\cdot 1\dot{3}7\dot{8}$, $\cdot 673\dot{2}0\dot{5}$, $\cdot 0\dot{7}178\dot{6}$ are mixed circulating decimals.

56. To reduce a pure repetend to its equivalent vulgar fraction:—

RULE.

Write the period itself for numerator, and for denominator as many 9's as there are digits in the period.

Thus, $\cdot\dot{7} = \frac{7}{9}$; $\cdot\dot{4}\dot{2} = \frac{42}{99} = \frac{14}{33}$; $\cdot\dot{7}1\dot{6} = \frac{716}{999}$.

$\cdot\dot{2}10\dot{7} = \frac{2107}{9999}$; $\cdot\dot{4}13\dot{1} = \frac{4131}{9999} = \frac{459}{1111}$.

57. To reduce a mixed repetend to its equivalent vulgar fraction:—

RULE.

Subtract the finite part of the mixed repetend from the whole, and write the remainder as numerator; then for denominator write as many 9's as there are digits in the circle, followed by as many 0's as there are digits in the finite part.

NOTE.—The finite part of a mixed repetend is the part between the decimal point and the first digit of the circle. The circle or repeater is the infinite part.

EXAMPLE.—Reduce $\cdot4\dot{1}\dot{7}$, $\cdot12\dot{3}\dot{4}$, and $\cdot716\dot{1}42\dot{3}$ to their equivalent vulgar fractions.

OPERATION.

$417 - 4 = 413$ = 1st numerator.
$1234 - 12 = 1222$ = 2d numerator.
$7161423 - 716 = 7160707$ = 3d numerator.

Also 1st decimal contains 2 digits in the circle and 1 in the finite part, therefore the denominator of the 1st fraction is 990.

Similarly the denominator of the 2d fraction is 9900, and of the 3d fraction 9999000.

Therefore $\cdot4\dot{1}\dot{7} = \frac{413}{990}$; $\cdot12\dot{3}\dot{4} = \frac{1222}{9900} = \frac{611}{4950}$; $\cdot716\dot{1}42\dot{3} = \frac{7160707}{9999000}$.

EXERCISE 51.

Express the following pure and mixed repetends as vulgar fractions:

1. $\cdot\dot{1}$; $\cdot\dot{7}\dot{8}$; and $\cdot\dot{4}$.
2. $\cdot\dot{2}\dot{1}$; $\cdot\dot{3}4\dot{7}$; $\cdot\dot{2}17\dot{8}$.
3. $\cdot\dot{1}986747\dot{3}$; $\cdot\dot{2}\dot{7}$.
4. $\cdot1\dot{2}\dot{6}$; $\cdot2\dot{1}\dot{4}$.
5. $\cdot21\dot{3}\dot{4}$, $\cdot21\dot{6}$; $\cdot21\dot{1}\dot{4}$.
6. $\cdot123\dot{4}\dot{5}$ $\cdot\dot{1}67\dot{8}$.
7. $\cdot671\dot{4}$; $\cdot12\dot{7}1\dot{6}$.
8. $\cdot91\dot{8}\dot{6}$; $\cdot1\dot{4}\dot{2}$.
9. $\cdot123\dot{4}\dot{7}$; $\cdot12\dot{7}\dot{8}$.
10. $\cdot16\dot{7}1\dot{4}$; $\cdot\dot{9}$; $\cdot\dot{8}\dot{6}$.
11. $27\cdot\dot{4}\dot{3}$; $17\cdot8\dot{1}\dot{6}$.
12. $467\cdot12\dot{3}4\dot{5}$; $16\cdot16\dot{1}6\dot{1}$.

---

58. In order to add, subtract, multiply, or divide pure or mixed repetends:—

RULE.

Reduce them to their equivalent vulgar fractions, and then add, subtract, multiply, or divide these fractions.

EXAMPLE 1.—From $9\cdot7\dot{6}$ take $4\cdot9\dot{1}\dot{8}$.

OPERATION.

$$9\cdot7\dot{6} = 9\tfrac{69}{90} = 9\tfrac{23}{30}, \text{ and } 4\cdot9\dot{1}\dot{8} = 4\tfrac{909}{990} = 4\tfrac{101}{110}.$$

$$\text{Then } 9\tfrac{23}{30} - 4\tfrac{101}{110} = 9\tfrac{253}{330} - 4\tfrac{303}{330} = 4\tfrac{280}{330} = 4\tfrac{28}{33}. \quad \textit{Ans.}$$

EXAMPLE 2.—Divide $\cdot9\dot{2}\dot{7}$ by $\cdot0123\dot{4}\dot{5}$.

OPERATION.

$$\cdot9\dot{2}\dot{7} = \tfrac{918}{990} = \tfrac{102}{110} = \tfrac{51}{55}, \text{ and } \cdot0123\dot{4}\dot{5} = \tfrac{12222}{99000} = \tfrac{679}{5500},$$

$$\text{Then } \tfrac{51}{55} \div \tfrac{679}{5500} = \tfrac{51}{55} \times \tfrac{5500}{679} = \tfrac{5100}{679} = 75\tfrac{75}{679}. \quad \textit{Ans.}$$

EXERCISE 52.

Find the value of—

1. $\cdot\dot{9} + \cdot\dot{6}5$.
2. $9\cdot\dot{1}\dot{2} + \cdot7\dot{2}\dot{5}$.
3. $6\cdot1\dot{4} - 2\cdot714$.
4. $7\cdot9\dot{1}8\dot{6} - 2\cdot3\dot{4}\dot{7}$.
5. $7\cdot\dot{5} + 1\cdot2\dot{3} + 7\cdot1\dot{9}1$.
6. $\cdot\dot{7} \times \cdot\dot{1}\dot{2} \times \cdot\dot{6}\dot{7}$.
7. $\cdot\dot{6}7 \times \cdot9\dot{1}\dot{4}$.
8. $6\cdot7\dot{1} \times 6\cdot7\dot{1}\dot{3}$.
9. $\cdot\dot{6}1\dot{4} + 2\cdot7\dot{6}\dot{6}$.
10. $1\cdot647 + 3\cdot5\dot{2}\dot{1}$.

---

59. To reduce a given denominate number to the decimal of another given number of a higher denomination:—

RULE.

Divide the lowest denomination named by that number which makes one of the next higher denomination.

Annex this quotient to the number of the next higher denomination given, and divide as before.

Proceed thus through all the denominations to the one required, and the last result will be the one sought.

EXAMPLE 1.—Reduce 2 pks. 1 qt. 1 pt. to the decimal of a bushel.

OPERATION.

2)1
4)1·5
2)0·375
4)2·1875
·546875

Here we first reduce the 1 pt. to the decimal of a qt. by dividing by 2; this gives us 1 pt. = ·5 of a qt., and to this we prefix the given quarts, 1. Then we divide the 1·5 quarts by 4 to reduce them to the decimal of a gallon, to which we prefix a 0, as no gallons are given; and so on.

60. Sometimes we must first reduce one quantity to a fraction of the other, and then reduce the resulting fraction to its decimal by Art. 44.

EXAMPLE 2.—Reduce 4s. 11¾d. to the decimal of £2 11s

Here, by Art. 25, $\frac{\text{4s. 11¾d.}}{\text{£2 11s.}} = \frac{\text{239 farthings}}{\text{2448 farthings}} = \frac{239}{2448}$;

Then 239 ÷ 2448 = ·09763. *Ans.*

EXERCISE 53.

Reduce :

1. 2 days 7 hrs. to the decimal of a week.
2. 7 oz. 4 dwt. 9 grs. to the decimal of a pound.
3. 16 lbs. 7 oz. 3 drs. to the decimal of a cwt.
4. 116 days 14 hours to the decimal of a year.
5. 1 rood 17 yards to the decimal of an acre.
6. 3 qrs. 1 na. 1 in. to the decimal of a French ell.
7. 16s. 11½d. to the decimal of a pound.
8. £9 14s. 8½d. to the decimal of £77 0s. 9d.
9. 2 days 17 min. to the decimal of 7 weeks 4 days.
10. 3 fur. 17 per. to the decimal of 2 miles 4 yds. 1 ft.
11. 17 lbs. 4 oz. to the decimal of 19 lbs. 7 oz. 5 drs. 1 scr.
12. 2 roods 27 yds. to the decimal of 29 per. 29 yds.

---

61. To find the value of a given decimal of a denominate number :—

RULE.

Multiply the given decimal by the number of units of the next lower denomination that make one of the given denomination.

Point off as many decimal places as there were in the multiplicand, and the integral portion, if any, will be units of that lower denomination. The decimal part may be reduced to a still lower denomination ; and so on.

EXAMPLE.—Find the value of ·27625 of a lb. Troy.

OPERATION.

·27625 = decimal of a lb.
12

3·31500 = oz. and decimal of an oz.
20

6·30000 = dwt. and decimal of a dwt.
24

7·20000 = grs. and decimal of a gr.

Then 3 oz. 6 dwt. 7·2 grs. *Ans.*

Here we first multiply by 12, because 12 oz. make 1 lb., and we thus get 3·31500 oz. Next we multiply the decimal ·31500 by 20 to reduce it to dwt., &c.

EXERCISE 54.

Find the value of—

1. ·146785 of £1.
2. ·71463 of a week.
3. 2·147 of a pound Apoth.
4. ·6143 of a mile.
5. ·916147 of an acre
6. 2·14617 of a French ell.
7. 9·2645 of an hour.
8. 4·7177 of a hhd.
9. 3·33625 of a rood.
10. 9·914 of £1.
11. 6·714 of £3 4s. 7½d.
12. 9·1467 of a year.
13. ·12345 of $2·78.
14. ·65265 of 27 sq. yds. 2 ft.
15. 7·46725 of 7 cwt. 2 qrs. 17 lbs.
16. 6·4715 of £7 7s. 7½d.

EXERCISE 55.

Miscellaneous Problems.

1. Reduce £297 4s. 3½d. to dollars and cents, and divide the result by ·0005.
2. Find the least common multiple of 9, 11, 18, 15, 21, 22, 42, 36, and 60.
3. Add together $78·90, $427·43, $209·17, $80·43, and $17·90, and from the sum subtract £183 15s. 11½d.
4. Reduce $\frac{1}{2}$, $\frac{4}{7}$, $\frac{3}{11}$, $\frac{2}{5}$, and $\frac{7}{10}$ to equivalent fractions having a common denominator.

5. How much is 726 times £2 4s. ·10¼d.?

6. How many times would a carriage wheel having a circumference of 13 ft. 7 in. revolve in going from Toronto to Stoney Creek, a distance of 44 miles?

7. Divide $7498·70 among A, B, and C, so that A shall have just $749·83 more than each of the others.

8. In 1858 the value of the horses imported into the different ports of Canada was as follows: Clifton $6880, Coaticook $9775, Morrisburgh $6750, Prescott $58877, Stanstead $9105, Toronto $8750, Windsor $9880, other ports $58712; what was the total value of the horses imported into Canada in 1858?

9. In the same year the value of the horned cattle imported into Canada was as follows: Coaticook $2702, Dundee $3537, Montreal $3830, Potton $2156, Sault Ste. Marie $3156, Windsor $10688, other ports $25561; how much did the value of the horses imported into Canada in the year 1858 exceed that of the horned cattle imported in the same year?

10. What is the difference between ⅓ of 3¼ of 5⅓ of $28·28 and ·7 of 2·4 of 3·$\dot{7}$ of ⅚ of £6 11s. 5½d.?

11. Express 704, 1111, 9876, 23471, and 9142371 in Roman numerals.

12. Write down as one number seven hundred billions four thousand and twenty, *and* six millions two hundred thousand and nineteen tenths of trillionths.

13. Find the value of $\frac{27 \times 45 \times 64 \times 117 \times 23 \times 115 \times 93 \times 144}{25 \times 729 \times 184 \times 27 \times 12 \times 13}$.

14. Find the value of 2¾ + ⅓ of ⅘ of $\frac{7}{11}$ of 37½ + ⅞ − ⅔ + 8⅓ − 4⅝.

15. Reduce 2 days 4 hours to the decimal of 3 weeks 3 days.

16. What is the Greatest Common Measure of 17810 and 83294?

17. Reduce ·$\dot{7}$, ·$\dot{9}\dot{3}$, ·$\dot{0}004\dot{5}$, and ·$27\dot{1}4\dot{0}$ to their equivalent vulgar fractions.

18. How many square inches are there in 2 a. 1 r. 17 per. 9 yds.?

19. What is the value of ·7149025 of a mile?

20. Find the value of ·7 of a per. + ·625 of a yd. + ·713 of a ft. + ·91 of an inch.

21. Which the greatest and which the least of $\frac{4}{15}$, $\frac{6}{25}$, $\frac{3}{11}$?

22. Express 3¼ ells Flem. as a fraction of a yard.

23. A farmer at a fair sold 229 sheep at $3·73 each, and bought 13 cows at £11 11s. 7d. each; how much money did he carry home?

24. In the year 1858 there were imported into Canada 20735 doz. eggs valued at $2487; in the same year there were exported from Canada 623525 doz. eggs valued at $66860; how many more eggs were exported than imported during that year, and what in each case was the average value per doz. eggs?

25. What is the l. c. m. of 6, 10, 16, 20, 24, 28, 32, 36, 40, and 44?

26. How many times 123 is 746 times 193?

27. The quotient is 794 and the divisor 83, what is the dividend?

28. Divide 749 lbs. 4 oz. 7 drs. Avoir. equally among 19 persons; what is the share of each?

29. Divide 346 a. 1 r. 17 per. by 2 a. 3 r. 27 per. 9 yds.

30. Reduce 2 bush. 1 pk. 1 qt. to the fraction of 11 bush. 3 pks.

31. Add together $947·60, $207·90, $274·33 and £217 4s. 7¼d., and divide the sum by 299.

32. Find the value of $\frac{63 \times 47 \times 21 \times 121 \times 264 \times 625}{35 \times 81 \times 55 \times 48 \times 517 \times 40}$

SECTION V.

RATIO, SIMPLE PROPORTION, COMPOUND PROPORTION.

RATIO.

1. The Ratio of one number to another is the *quotient* arising from the division of the former by the latter.

Thus, the ratio of 16 to 4 is $16 \div 4 = 4$.
the ratio of 27 to 8 is $27 \div 8 = 3\frac{3}{8}$, &c.

2. The Ratio of one number to another is commonly expressed by writing a colon between them.

Thus, the ratio of 16 to 4 is expressed by 16 : 4.
the ratio of 27 to 8 is expressed by 27 : 8.

3. The Ratio of one number to another may also be expressed by writing them in the form of a fraction.

Thus, the ratio of 16 to 4 may be expressed by $\frac{16}{4}$.
the ratio of 27 to 8 may be expressed by $\frac{27}{8}$.

4. The two numbers that constitute the ratio are called the terms of the ratio; and the first term is called the *antecedent* and the other the *consequent* of the ratio.

5. If the antecedent is equal to the consequent, the ratio is called a *ratio of equality.*

If the antecedent is greater than the consequent, the ratio is called a *ratio of greater inequality.*

If the antecedent is less than the consequent, the ratio is called a *ratio of less inequality.*

6. A Simple Ratio is the ratio of any one number to any other number.

7. A Compound Ratio is a ratio produced by compounding or multiplying together the corresponding terms of two or more simple ratios.

8. The value of a ratio is found by dividing the antecedent by the consequent.

9. Ratios are compared together by comparing their values together.

10. Ratios are compounded together by multiplying together all the antecedents for a new antecedent and all the consequents for a new consequent.

Note.—Before multiplying the antecedents together for a new antecedent and the consequents together for a new consequent we cancel any factor that is common to an antecedent and a consequent.

Example 1.—What is the ratio of 27 to 3?

OPERATION.

$27 \div 3 = 9.$ *Ans.*

Example 2.—What is the ratio of 83 to 6?

OPERATION.

$83 \div 6 = 13\frac{5}{6}.$ *Ans.*

Example 3.—What is the value of the ratio of 94 to 7?

OPERATION.

$94 \div 7 = 13{\cdot}428.$ *Ans:*

Example 4.—What is the value of the ratio of 17 to 23?

OPERATION.

$$17 \div 23 = 0{\cdot}739. \quad \textit{Ans.}$$

Example 5.—Point out which is the greatest and which the least of the following ratios—7 : 4, 9½ : 5, 27 : 16.

OPERATION.

$$\begin{aligned} 7 : 4 &= 7 \div 4 = 1{\cdot}75. \\ 9\tfrac{1}{2} : 5 &= 9{\cdot}5 \div 5 = 1{\cdot}9. \\ 27 : 16 &= 27 \div 16 = 1{\cdot}68. \end{aligned}$$

Therefore 9½ : 5 is the greatest and 27 : 16 the least.

Example 6.—Find the ratio compounded of 6 : 7, 9 : 4, 11 : 13, 12 : 55, and 5 : 27.

STATEMENT. CANCELLED.

$$\frac{6}{7} \times \frac{9}{4} \times \frac{11}{13} \times \frac{12}{55} \times \frac{5}{27} = \frac{6}{7} \times \frac{\cancel{9}}{\cancel{4}} \times \frac{\cancel{11}}{13} \times \frac{\overset{4}{\cancel{12}}}{\underset{\cancel{11}}{\cancel{55}}} \times \frac{\cancel{5}}{\underset{\cancel{3}}{\cancel{27}}} =$$

$$\frac{6}{7 \times 13} = \tfrac{6}{91} = 6 : 91. \quad \textit{Ans.}$$

Exercise 56.

1. What is the ratio of 7 : 4? 19 : 3? 26 : 2? 144 : 6? 29 : 2? 16 . 3?
2. What is the ratio of 27 : 4⅓? 11 : 19? 26 . 7? 28 : 7? 129 : 2? 47 : 18?

Find the value of—

3. 7 : 2; 9 : 14; 63 : 7; 29 : 3; 19 : 27; 34 : 6.
4. 91 : 7; 16 : 3; 28 : 5; 111 : 7; 222 : 11; 167 : 29.

Compare together the following ratios, and point out which is the least and which the greatest:

5. 9 : 17, 16 : 33, and 47 : 79.
6. 11 : 3, 17 : 5, 38 : 11, and 164 : 55.
7. 49 : 5, 176 : 16·4, 267·4 : 25·9, and 8 : ·89.

Compound together the following ratios:

8. 7 : 4, 11 : 23, 11⅓ : 9, and 9 : 14.
9. 6 : 11, 12 : 17, 8¼ : 4⅓, 27 : 121, and 5⅓ : 6.
10. 15 : 4, 16 : 7, 9 : 20, 70 · 27, and 6 : 5.
11. 8 : 7, 6 : 5, 4 . 3, 2 : 1, and 21 . 32.
12. 2 : 3, 4 : 5, 6 : 7, 8 : 9, and 16 : 23.

SIMPLE PROPORTION.

11. Simple Proportion enables us to find a fourth number which shall have the same ratio to the third of three given numbers that the second of these numbers has to the first: hence proportion consists in an equality of ratios.

12. In every simple proportion three terms are given to find the fourth, and this fourth term must be of the same name or denomination as the third.

13. Proportion is expressed by writing the sign :: between the two equal ratios that compose the proportion.

Thus, the proportion existing between 7, 21, 19, and 57 is expressed thus—7 : 21 : : 19 : 57, and is read 7 is to 21 as 19 is to 57.

14. The two outer terms of a proportion are called the extremes, and the two intermediate terms the means.

Thus, in the above example 7 and 57 are the extremes, and 21 and 19 the means.

15. In every proportion the product of the extremes is equal to the product of the means.

us, in the following examples we have

9 : 54 :: 2 : 12, and 9 × 12 = 54 × 2.
7 : 21 :: 19 : 57, and 7 × 57 = 21 × 19.
16 : 3 :: 12 : 2$\frac{1}{4}$, and 16 × 2$\frac{1}{4}$ = 3 × 12.

16. Since *1st term* × *4th term* = *2d term* × *3d term*, it follows that the *4th term* = $\frac{\textit{2d term} \times \textit{3d term}}{\textit{1st term}}$.

That is, the 4th term of every proportion is found by multiplying together the 2d and 3d terms and dividing their product by the first term.

Example 1.—What is the fourth proportional to 7, 11, and 23?

OPERATION.

7 : 11 :: 23 : *Ans.* Hence *4th term* or *Ans.* = $\frac{11 \times 23}{7}$ = 36$\frac{1}{7}$.

EXAMPLE 2.—Find a fourth proportional to 24, 105, and 40.

OPERATION.

$$24 : 105 :: 40 : Ans. \quad \text{Hence } Ans. = \frac{105 \times 40}{24} = \frac{\overset{35}{\cancel{105}} \times \overset{5}{\cancel{40}}}{\underset{\cancel{3}}{\cancel{24}}}$$

$$= 35 \times 5 = 175.$$

EXAMPLE 3.—Find a fourth proportional to 30, 76, and 99.

OPERATION.

$$30 : 76 :: 99 : Ans. \quad \text{Hence } Ans. = \frac{\overset{38}{\cancel{76}} \times \overset{33}{\cancel{99}}}{\underset{\underset{5}{\cancel{10}}}{\cancel{30}}} = \frac{38 \times 33}{5} = 250\tfrac{1}{5}.$$

EXERCISE 57.

Find the fourth proportional to the following numbers :

1. 7, 21, and 40.
2. 91, 7, and 46.
3. 11, 3, and 17.
4. 9, 47, and 29.
5. 6, 23, and 42.
6. 111, 21, and $18\frac{1}{4}$.
7. 9, 10, and 11.
8. 13, 14, and 65.
9. 1728, 109, and 72.
10. 253, 16, and 11.
11. 9, 891, and 100.
12. 9 days, 21 days, and $61·50.
13. 11 lbs., 147 lbs., and £16 4s. $11\frac{1}{2}$d.
14. 3 cwt., 20 cwt., and $66·87.
15. 9 miles, 17 miles, and 16 days.
16. 21 acres, 47 acres, and 11 wks.
17. 17 bushels, 29 bushels, and £6 7s. 4d.
18. 211 acres, 1 acre, and $7496·40.
19. 62 miles, 3 miles, and $421·40.
20. 7 months, 23 months, and $56·70.

---

17. In the preceding exercise the first three terms of the proportion are given in their proper order, but very generally in proportion the pupil is required to make the statement himself, and this is done according to the following :—

RULE.

I. *Reduce the two numbers which are of different names from the answer to the lowest denomination contained in either of them.*

II. *Set the number which is of the same kind as the answer in the third place, and, when the answer is to be greater than this third term, write the greater of the other two numbers in the second place; but when the answer is to be less than the third term, write the smaller of the other two numbers in the second place.*

III. *Multiply the second and third terms together, and divide their product by the first term.*

PROOF.—*Multiply the answer by the first term, and the product should be the same as that obtained by multiplying together the second and third terms.*

EXAMPLE 1.—If 11 men can mow a field in 23 days, in what time will 5 men mow it?

OPERATION.

5 : 11 : : 23 : *Ans.*
11
5)253
$50\frac{3}{5}$ days.

Here, as the answer must be days, we place the 23 days as the 3d term. Then, since it is evident that 5 men will require more days to mow the field than 11 men, that is, since the answer must be greater than the third term, we place 11, the greater of the two remaining numbers, as the second term.

EXAMPLE 2.—If 16 bushels of oats cost $6·70, what will 96 bushels cost?

OPERATION.

$$\cancel{16} : \overset{6}{\cancel{96}} : : \$6{\cdot}70 : \textit{Ans.}$$
6
$40·20 *Ans.*

Here, since the answer must be money, we put the $6·70 in the third place; then, because the answer must evidently be greater than the third term, 96 bushels costing more than 16 bushels, we put 96, the greater of the two remaining numbers, in the second place. Lastly, before applying the rule we cancel by dividing the first and second terms by 16.

EXAMPLE 3.—If 9 acres of grass will pasture 21 cows, how many cows will 6 acres pasture?

OPERATION.

$$\overset{3}{\cancel{9}} : \overset{2}{\cancel{6}} : : \overset{7}{\cancel{21}} : Ans.$$

$$7 \times 2 = 14. \ Ans.$$

Here we put 21, the number of cows, in the third term, since the answer is to be cows; next, since the answer will be less than the third term, 6 acres pasturing fewer cows than 9 acres, we place 6, the smaller of the two remaining terms, in the second place.

EXAMPLE 4.—If 6 lbs. 4 oz. 7 drs. Avoir. cost $169·40, what will 1 lb. 11 oz. Avoir. cost?

OPERATION.

6 lbs. 4 oz. 7 drs. = 1607 drs.
1 lb. 11 oz. = 432 drs.
1607 : 432 : : $169·40 : *Ans.*

```
                432
              33880
             50820
            67760
1607)73180·80($45·538. Ans.
     6428
      8900
      8035
       865·8
       803·5
        62·30
        48·21
        14·090
        12·856
         1·234
```

EXAMPLE 5.—If £2 6s. 8d. pay for 17 days' work, for how many days will £9 11s. 4d. pay?

OPERATION.

$$\begin{array}{l}
\pounds 2\ \ 6s.\ \ 8d. = \ \ 560d. \\
\pounds 9\ \ 11s.\ \ 4d. = 2296d. \\
\ \ 10\ \ \ \ \ \ 41 \\
\ \ \cancel{70}\ \ \ \ \cancel{287} \\
\cancel{560} : \cancel{2296} : : 17 : Ans.
\end{array}$$

$$\begin{array}{r}
41 \\
\underline{17} \\
68 \\
10\overline{)697} \\
\hline
69\tfrac{7}{10}\text{ days. } Ans.
\end{array}$$

EXAMPLE 6.—If $3\frac{2}{7}$ barrels of apples pay for $9\frac{3}{11}$ bushels of wheat, how many bushels of wheat can be purchased for $17\frac{4}{5}$ barrels of apples?

OPERATION.

$$3\tfrac{2}{7} : 17\tfrac{4}{5} : : 9\tfrac{3}{11} : Ans.$$

$$\text{or } \tfrac{23}{7} : \tfrac{89}{5} : : \tfrac{102}{11} : Ans.$$

$$\text{Hence the } Ans. = \tfrac{89}{5} \times \tfrac{102}{11} \div \tfrac{23}{7}$$

$$\text{or } \tfrac{89}{5} \times \tfrac{102}{11} \times \tfrac{7}{23} = \tfrac{63546}{1265} = 50\tfrac{296}{1265}.\ Ans.$$

Here, after making the statement, we reduce the terms to their equivalent improper fractions, invert the first term or divisor and connect it to the other two by the sign of multiplication.

EXAMPLE 7.—If $4\frac{2}{5}$ days' work cost $21\frac{3}{5}$ shillings, what will $17\frac{6}{11}$ days' work cost?

OPERATION.

$$4\tfrac{2}{5} : 17\tfrac{6}{11} : : 21\tfrac{3}{5} : Ans.$$

$$\text{or } \tfrac{22}{5} : \tfrac{193}{11} : : \tfrac{108}{5} : Ans.$$

$$\text{Hence } Ans. = \frac{\cancel{5}}{\underset{11}{\cancel{22}}} \times \frac{193}{11} \times \frac{\overset{54}{\cancel{108}}}{\cancel{5}} = \frac{193 \times 54}{11 \times 11} = 86\tfrac{16}{121}s.$$

EXERCISE 58.

1. If 28 men can dig 27 acres in a week, how many acres can 42 men dig in the same space of time?

2. What will 65 lbs. of sugar cost if $1·30 pay for 13 lbs.?

3. How many men would perform in 125 days a piece of work which 100 men can perform in 145 days?

4. If a person can finish a journey in 100 days, travelling 10 hours per day, how many days would he take to do it if he travelled only 6 hours per day?

5. If 13⅓ yards of cloth cost $12·90, what will $4\frac{3}{16}$ yards cost?

6. A bankrupt's effects amounted to $7149, which paid his creditors 80 cents in the $; to what sum did his debts amount?

7. What will be the cost of 16714 feet of clear timber at $18·70 per thousand?

8. What will be the tax on $7149·70 at the rate of 1¼ cents on the $?

9. If $\frac{3}{7}$ of a person's income is $7194·60, what is his income?

10. What is the cost of 127 acres of land if 871 acres cost $8671·40?

11. If 702 yds. of muslin cost £48 2s. 4½d., what will 540 yds. cost?

12. If a ship has water sufficient to last a crew of 35 men for 6 months, how long will it last a crew of 20 men?

13. What is the tax on £1749 16s. 8¼d. at 3s. 4d. in the £?

14. If 24 a. 1 r. 17 per. cost $763·80, what will 7 a. 1 per. 9 yds. cost?

15. If the railway fare for 100 miles is $3·75, what ought it to be for 63 miles?

16. How much must be given for 276 bushels of wheat at the rate of $7·90 for 7 bushels?

17. A bankrupt's debts amount to $71911·40 and his effects to $53069·80; how much can he pay in the $?

18. What will be the cost of draining 247 a. 1 r. 27 per. when 17 a. 1 r. 36 per. cost £111 17s. 8d?

19. If 16 barrels of flour can be bought for $97·80, what must be paid for 27 barrels?

20. There are two numbers in the proportion of 7 to 8 and the larger number is 291; what is the smaller?

21. What will be the cost of 71 cwt. 1 qr. 17 lbs., if $21·60 pay for 3 cwt. 2 qr. 26 lbs.? (Allowing 112 lbs. to the cwt.)

22. If 3¼ yds. of linen cost $2·21, what will 7⅜ yds. cost?

23. A besieged town containing 15000 inhabitants has provisions sufficient to last 5 weeks; how long will the provisions last if 7000 of the inhabitants be sent away?

24. If a stick 7 feet high cast a shadow 5 feet in length, what is the height of a spire that casts a shadow 129 feet in length?

25. How far can a man travel in 27 days at the rate of 149 miles in 4 days?

26. If a person steps over 4 yds. in 5 paces, to how many yards will 729 of his paces be equal?

27. There are two numbers in the proportion of 6 to 11 and the smaller is 29, what is the larger?

28. At 29 cents per lb., what will be the cost of 174 lbs. of raisins?

29. How much land at \$4·75 per acre must be given in exchange for 243 acres at \$3·60 per acre?

30. If $4\frac{3}{7}$ lbs. of nutmegs cost £$\frac{10}{21}$, what will $27\frac{3}{8}$ lbs., cost?

31. If $6\frac{1}{4}$ acres of land cost \$$67\frac{4}{11}$, for how much land will \$$23\frac{9}{13}$ pay?

32. If 4·32 lbs. of coffee cost \$1·17, what will 9·78 lbs. cost?

33. What will $9\frac{3}{7}$ lbs. of spice cost when \$$17\frac{3}{8}$ pays for 19·87 lbs.?

34. If 11 cows make 29 pounds of butter per week, how much may be expected from 27 cows?

35. If 7 men put up 200 perches of fencing in 2 days, how long would they take to put up 900 perches?

36. If \$100 stock is worth \$$95\frac{7}{11}$, how much can be purchased for \$100?

37. What will 16 lbs. 4 oz. 2 dwt. cost if 11 oz. 11 dwt. 11 grs. cost \$47·90?

38. If the rent of 73 a. 14 per. be £17 4s. 9d., what will be the rent of 33 a. 1 r. 23 per.?

39. If $\frac{1}{2}$ of $\frac{3}{7}$ of $\frac{3}{8}$ of $17\frac{1}{4}$ lbs. cost $\frac{3}{7}$ of $\frac{3}{11}$ of \$38·50, what will $6\frac{1}{4}$ of $\frac{1}{3}$ of $\frac{3}{7}$ of $8\frac{1}{4}$ lbs. cost?

40. Two numbers are to one another as 11 to 20, and the greater of the two is 107, what is the smaller?

COMPOUND PROPORTION.

18. Compound Proportion is an equality between a *compound* ratio and a *simple* ratio.

19. Compound Proportion is also called the Double Rule of Three, because all questions in compound proportion, when worked by simple proportion, require *two* or more statements.

20. In questions in Compound Proportion, five or more terms are given to find another term; that is, three or more ratios are given, one of which is imperfect or wants a term.

21. Questions in Compound Proportion are solved by the followi[illegible]:—

RULE.

Write, in the third place, that term which is of the same kind as the answer.

Of the other quantities, take each pair of corresponding terms, and, having first reduced both to the same denomination, arrange them as in simple proportion.

Then multiply together the third term and all the second terms, and divide the result by the product of all the first terms. The quotient will be the answer, in the same denomination as the third term.

Note.—Before thus multiplying and dividing, be careful to reduce, by cancelling, all the terms as much as possible.

Example 1.—If 10 men in 7 days can earn $78·43, how much can 17 men earn in 3 days?

OPERATION.

```
10 : 17 |
 7 :  3 | :: $78·43 : Ans.
              17
           54901
           7843
          1333·31
                3
   70)3999·93($57·1418
      350
       499
       490
         9 9
         7·0
         2·93
         2·80
          ·130
            70
           600
           560
            40
```

Here we set $78·43 in the 3d term, because the answer is to be money. Next we take the other terms in pairs, first for instance, the numbers referring to men, and of these we place the greater in the second place, because, leaving the time out of consideration, and regarding only the men and the wages, it is evident that the answer must be greater than the third term. Again, for a similar reason, leaving the men out of consideration, we place 3, the least of the remaining terms, in the second place. Lastly, we multiply together the $78·43, the 17, and the 3, and divide their product by 70, which is the product of the two first terms.

Example 2.—If 20 men can build 405 yds. of wall in 2 days, how many men will it require to build 522 yds. in days?

OPERATION.

STATEMENT.

$$\left.\begin{array}{r}405 : 522\\ 8 : 27\end{array}\right| :: 20 : \mathit{Ans.}$$

CANCELLED.

$$\begin{array}{rcl} \cancel{9} & & 29 \\ \cancel{81} & & \cancel{58} \\ \cancel{405} & : & \cancel{522} \\ \cancel{8} & : & \cancel{27} \\ \cancel{2} & & 3 \end{array} :: \overset{\cancel{4}}{\cancel{20}}$$

Then $3 \times 29 = 87$. *Ans.*

Here, since the answer is to be men, we place 20 men in the third term. Next, we take the two numbers referring to length of wall, and, leaving the time out of consideration, arrange these as in simple proportion. Afterwards, we take the two numbers referring to time, and, leaving length of wall out of consideration, also arrange these as in simple proportion.

Exercise 59.

1 If 24 men can dig 7 acres in 12 days, how many acres can 17 men dig in 22 days?

2. If a family of 11 people spend $490 in 7 months, how much will a family of 7 persons spend in 16 months?

3. If 110 reams of paper make 5000 copies of a book of 15 sheets, how much paper will be required for 4000 copies of a book of 11 sheets?

4. If 21 men can mow 93 acres in 5 days, how long will 7 men require to mow 16 a. 3 r. 20 per.?

5. If 50 men can dig an excavation in 7 days, working 11 hours per day, how many days will 24 men require when they work only 8 hours per day?

6. If $750 gain $204 in 23 months, how much will $467 gain in 7 months?

7. If a wall 79 feet long, 4 feet high, and 2 feet thick be built by 17 men in 11 days, what length of wall 5 feet high and 3 feet thick can be built by 34 men in 33 days?

8. If 3 men can cradle 34 acres of wheat in 5 days, how many men will it require to cradle 85 a. 32 per. in 6 days?

9. If a ditch 36 feet long, 8 feet deep, and 4 feet wide be dug by 32 men in 4 days, in what time will 48 men dig a ditch 84 feet long, 6 feet deep, and 8 feet wide?

10. If 34 men can saw 90 cords of wood in 6 days when the days are 9 hours long, how many cords can 8 men saw in 36 days when the days are 12 hours long?

11. If 5 compositors in 16 days, each 11 hours long, can compose 25 sheets of 24 pages in each sheet, 44 lines in a page, and 40 letters in a line, in how many days, each 10 hours long, can 9 compositors compose 36 sheets of 16 pages to a sheet, 50 lines to a page, and 45 letters to a line?

12. If 243 men in 5½ days of 11 hours each, dig a trench of 7 degrees of hardness, 232½ yards long, 3⅔ yards wide, and 2⅓ yards deep, in how many days, of 9 hours each, will 24 men dig a trench of 4 degrees of hardness, 337½ yards long, 5⅜ yards wide, and 3⅓ yards deep?

13. If 60 men can dig a trench 500 feet long, 36 wide, and 40 deep, in 24 days of 8 hours each, how many men will be required to dig a trench 550 feet long, 68 wide, and 90 deep, in 44 days of 9 hours each?

14. If 9 lbs. 6 oz. 4 dwt. of silver make 5 dozen forks, each worth 11s. 4½d., how many forks, each worth 7s. 8¼d., can be made out of 11 lbs. 11 oz. 17 grs.?

15. If 279 bushels of potatoes feed 4 cows for 60 days, how many bushels will be required to feed 27 cows for 200 days?

16. If 7·3 acres of land are trenched by 23 men in 27·9 days, working 11·4 hours per day, how many acres of land may be trenched by 48 men in 16·5 days when they work 9·4 hours per day?

17. If the wages of 11 men for 11 days be $111·11, what will be the wages of 16 men for 16 days?

18. If a block of marble 8 feet long, 4 feet wide, and 2 feet thick, weigh 8550 lbs., what will be the weight of another block of marble 6 feet long, 6 feet wide, and 4 feet thick?

19. If a rectangular vat 8 feet square and 2½ feet deep hold 10000 lbs. of water, how many pounds of water will a rectangular vat 10 feet long, 8 feet wide, and 2 feet deep, contain?

20. If 14 oz. of wool make 2¼ yds. of flannel 1⅜ yds. wide, how many ounces of wool will be required to make 17⅗ yds. of flannel 1³⁄₇ yds. wide?

21. If 2043³⁄₇ yds. of cloth 1⅜ yds. wide make coats for a regiment of soldiers containing 847 men, how much cloth 2¼ yds. wide will be required to make coats for another regiment which contains 981 men?

22. If 8 men can cradle 97 acres in 4 days of 7⅗ hours each, how many acres will 14 men cradle in 3¼ days of 9⅜ hours each?

23. If $450 gain $24 in 12 months, what principal will gain $97 in 4 months?

24. If 24 horses eat 54 bushels of oats in 9 days, how many bushels of oats will last 29 horses 27 days?

SECTION VI.

PRACTICE.

1. Practice is a short method of finding the value of any quantity of merchandise, the value of a unit of any denomination being given.

2. An Aliquot Part is an exact or even part.

Thus 20 cents is an aliquot or even part of $1 ; 2 oz. is an aliquot part of 1 lb. ; 6 months, 4 months, 3 months, 2 months, $1\frac{1}{2}$ months, and 1 month, are aliquot parts of 1 year.

TABLE OF ALIQUOT PARTS.

Parts of $1.	Parts of a month.	Parts of £1.	Parts of 1s.	Parts of a cwt.* of 112 lbs.
50 cts. = $\frac{1}{2}$	15 days = $\frac{1}{2}$	10s. = $\frac{1}{2}$	6d. = $\frac{1}{2}$	56 lbs. = $\frac{1}{2}$
$33\frac{1}{3}$ = $\frac{1}{3}$	10 = $\frac{1}{3}$	6s. 8d. = $\frac{1}{3}$	4d. = $\frac{1}{3}$	28 lbs. = $\frac{1}{4}$
25 = $\frac{1}{4}$	$7\frac{1}{2}$ = $\frac{1}{4}$	5s. = $\frac{1}{4}$	3d. = $\frac{1}{4}$	16 lbs. = $\frac{1}{7}$
20 = $\frac{1}{5}$	5 = $\frac{1}{6}$	4s. = $\frac{1}{5}$	2d. = $\frac{1}{6}$	14 lbs. = $\frac{1}{8}$
$16\frac{2}{3}$ = $\frac{1}{6}$	3 = $\frac{1}{10}$	3s. 4d. = $\frac{1}{6}$	$1\frac{1}{2}$d. = $\frac{1}{8}$	8 lbs. = $\frac{1}{14}$
$12\frac{1}{2}$ = $\frac{1}{8}$	2 = $\frac{1}{15}$	2s. 6d. = $\frac{1}{8}$	1d. = $\frac{1}{12}$	7 lbs. = $\frac{1}{16}$
$8\frac{1}{3}$ = $\frac{1}{12}$	1 = $\frac{1}{30}$	2s. = $\frac{1}{10}$		
$6\frac{1}{4}$ = $\frac{1}{16}$		1s. 8d. = $\frac{1}{12}$		Parts of a qr. of 28 lbs.
5 = $\frac{1}{20}$	NOTE.—The aliquot parts of a year are the same as those of a shilling.—See 4th column.	1s. 4d. = $\frac{1}{15}$		
4 = $\frac{1}{25}$		1s. 3d. = $\frac{1}{16}$		14 lbs. = $\frac{1}{2}$
2 = $\frac{1}{50}$		1s. = $\frac{1}{20}$		7 lbs. = $\frac{1}{4}$
				$3\frac{1}{2}$ lbs. = $\frac{1}{8}$
				$1\frac{3}{4}$ lbs. = $\frac{1}{16}$

---

* Although we allow but 100 lbs. to the cwt. in Canada, it is often necessary to make calculations with the old cwt. of 112 lbs. This arises from the fact that the latter is still in common use in Great Britain, several of the States of the American Union, &c. The aliquot parts of the new cwt. of 100 lbs. are the same as the aliquot parts of $1.

Example 1.—What is the cost of 47 cows at $33·40 each?

OPERATION.

$33·40 × 47 = $1569·80. *Ans.*

Example 2.—What is the value of 1678 lbs. of tea at $1·50 per lb.?

50 cts.	½	$1678	= value of 1678	@ $1.
		839	= “ “	@ ·50
		$2517	= “ “	@ $1·50

Example 3.—Find the price of 2164 articles at $1·87½ each.

OPERATION.

50 cts.	½	$2164	= price of 2164 articles	@ $1	each.
25	½	1082	= “ “ “	@ ·50	“
12½	½	541	= “ “ “	@ ·25	“
		270·50	= “ “ “	@ ·12½	“
		$4057·50	= “ “ “	@ $1·87½	“

Example 4.—Find the value of 978 sheep at $3·79 each.

OPERATION.

50 cts.	½	$978	= value of 978 sheep	@ $1	each.
20 cts.	⅕	3			
		$2934	= “ “	@ $3	“
		489	= “ “	@ ·50	“
5 cts.	¼	195·60	= “ “	@ ·20	“
4 cts.	⅕	48·90	= “ “	@ ·05	“
		39·12	= “ “	@ ·04	“
		$3706·62	= “ “	@ $3·79	“

Note.—In all questions similar to the preceding, where the value of one article is given to find that of a certain number of articles of the same denomination, the shortest and simplest mode of proceeding is to multiply the price of *one* article by the given number of articles. Thus:—

Example 2.—*Ans.* = $1·50 × 1678.
Example 3.—*Ans.* = $1.87½ × 2164.
Example 4.—*Ans.* = $3·79 × 978.

EXAMPLE 5.—Find the value of 1679 lbs. 14 oz. 12 drs. Avoir. at $169·40 per lb.

OPERATION.

8 oz.	½	$169·40 × 1679 =	$284422·60 =	value of 1679 lbs.
4 oz.	½	84·70	156·164 =	" " 14 oz. 12 drs.
2 oz.	½	42·35	$284578·76⅖ =	" 1679 lbs. 14 oz. 12 drs.
8 drs.	¼	21·175		
4 drs.	½	5·293		
		2·646		
		$156·164		

EXAMPLE 6.—Allowing 112 lbs. to the cwt., find the value of 229 cwt. 3 qrs. 17 lbs. of tallow at $6·20 per cwt.

OPERATION.

2 qr.	½	$6·20 × 229 =		$1419·80 =	value of 229 cwt.
1 qr.	½	3·10 =	value of 2 qr.	5·59 =	" " 3 qr. 17 lb.
14 lb.	½	1·55 =	" of 1 qr.	$1425·39 =	" 229 cwt. 3 qr. 17 lb.
2 lb.	$\frac{1}{7}$	·775 =	" of 14 lb.		
1 lb.	½	1107 =	" of 2 lb.		
		·0553 =	" of 1 lb.		
		$5·591			

EXAMPLE 7.—What is the value of 29 lbs. 7 dwt. 10 grs. of gold at £3 17s. 11¼d. per oz.?

OPERATION.

29 lbs. = 348 ounces.

5 dwt.	¼	£3 17s. 11¼d. × 348 = £1356 2s. 3d. = value of 29 lbs.				
2½ dwt.	½		19s.	$5\frac{13}{16}$	=	value of 5 dwt.
6 grs.	$\frac{1}{10}$		9s.	$8\frac{29}{32}$	=	" 2 dwt. 12 grs.
1 gr.	$\frac{1}{6}$			$11\frac{221}{320}$	=	" 6 grs.
				$1\frac{1821}{1920}$	=	" 1 gr.
		£1	10s.	$4\frac{229}{640}$	=	" 7 dwt. 19 grs.

Then	£1356	2s.	3d.	=	value of 29 lbs.
	1	10s.	$4\frac{229}{640}$d.	=	" 7 dwt. 19 grs.
	£1357	12s.	$7\frac{229}{640}$d.	=	" 29 lbs. 7 dwt. 19 grs.

EXAMPLE 8.—What is the price o 7149 tons of hay at £2 13s. 9d. per ton?

OPERATION.

10s.	½	£7149 2		=	value of 7149 tons	@	£1	per ton.
		£14298		=	" "	@	£2	"
2s. 6d.	¼	3574	10s.	=	" "	@	10s.	"
1s. 3d.	½	893	12s. 6d.	=	" "	@	2s. 6d.	"
		446	16s. 3d.	=	" "	@	1s. 3d.	"
		£19212	18s. 9d.	=	" "	@	£2 13s. 9d.	"

EXAMPLE 9.—Find the price of 7149⅜ acres of land at $27·43 per acre.

OPERATION.

$27·43 × 7149 = $196097·07 = value of 1149 acres.
$27·43 × 3 ÷ 8 = 10·28⅝ = " " ⅜ of an a.
$196107·35⅝ = " " 7149⅜ acres.

EXERCISE 60.

Find the value of—

1. 229 at $2·75.
2. 743 at $3·81.
3. 7114 at $97·86¼.
4. 213 at £2 16s. 4d.
5. 321 at £9 1s. 1½d.
6. 7147 at £12 12s. 2¾d.
7. 217¾ at $914·70.
8. 618$\frac{4}{9}$ at $42·71¼.
9. 907$\frac{11}{12}$ at $16·93.
10. 204¼ at £2 7s. 8¾d.
11. 604$\frac{3}{7}$ at £93 13s. 7d.
12. 904$\frac{37}{45}$ at £16 4s. 9¾d.
13. 617 lbs. 4 oz. Avoir. at $91·43 per lb.
14. 2171 a. 2 r. 17 per. at $9·70 per acre.
15. 114 bush. 1 pk. 1 gal. 1 qt. at 37½ cents per bushel.
16. 209 lbs. 7 dwt. 16 grs. at $1·71 per oz.
17. 614 yds. 2 qrs. 1 na. at $2·73 per yd.
18. 16 a. 1 r. 4 per. 7 yds. at £2 17s. 6d. per acre.
19. 29 wks. 4 days 11 h. at $7·40 per week.
20. 167 miles 7 fur. 6 per. at £9 3s. 6d. per mile.
21. 217 lbs. 4 oz. 6 drs. 2 scr. at £9 6s. 7d. per oz.
22. 9167 sheep at £1 3s. 6d. each.
23. 21791 bushels of wheat at $1·40 per bushel.
24. 1673$\frac{3}{11}$ sq. yds. of painting at 2s. 3¼d. per sq. yd.
25. 437 a. 9 per. 7 yds. of land at $21·40 per acre.
26. 97 cub. yds. 4 ft. at $0·73 per cub. yd.
27. 614$\frac{3}{19}$ cwt. of iron at $1·23 per cwt.
28. 23 lbs. 4 oz. 7 dwt. 11 grs. at 11¾d. per dwt.
29. 216 cwt. 2 qrs. 19 lbs. at $96·71 per cwt.*
30. 179 cwt. 1 qr. 23 lbs. at £9 14s. 11¾d. per cwt.*

---

* Allowing 112 lbs. to the cwt.

SECTION VII.

PERCENTAGE, COMMISSION, BROKERAGE INSURANCE, STOCKS.

PERCENTAGE.

1. Percentage or Per Cent. means a certain allowance or rate per 100. Per Cent. is a contraction of the Latin *per centum*, and means, "by the hundred."

Thus, if a person purchase 100 barrels of flour and some of them become worthless through being damaged, he is said to have lost 1 per cent., 2 per cent., 3 per cent., 7 per cent., 15 per cent., or 29 per cent., &c., of his flour, according as his loss is 1, 2, 3, 7, 15, or 29 barrels.

2. When the rate per cent. is given, the rate per unit is found by dividing by 100, or what amounts to the same thing, removing the decimal point two places to the left in the number that expresses the rate per cent.

Example 1.—What rate per unit is equivalent to 6 per cent.?

Ans. $6 \div 100 = ·06$.

Example 2.—What rate per unit is equivalent to $8\frac{3}{4}$ per cent.?

Ans. $8\frac{3}{4} \div 100 = 8·75 \div 100 = ·0875$.

Example 3.—What rate per unit is equivalent to 23 per cent.?

Ans. $23 \div 100 = ·23$.

Example 4.—What rate per cent. is equivalent to ·243 per unit?

Ans. $·243 \times 100 = 24\frac{3}{10}$.

Example 5.—What rate per cent. is equivalent to ·075 per unit?

Ans. $·075 \times 100 = 7·5 = 7\frac{1}{2}$.

Example 6.—What rate per unit is equivalent to 12·63 per cent.?

Ans. 12·63 ÷ 100 = ·1263.

Exercise 61.

What rate per unit is equivalent to—

1. 9 per cent.? $4\frac{1}{3}$ per cent.?
2. 3·7 per cent.? $29\frac{1}{4}$ per cent.?
3. 6·2 per cent.? $8\frac{1}{8}$ per cent.?
4. 111 per cent.? 147 per cent.?
5. $9\frac{1}{4}$ per cent.? $63\frac{4}{25}$ per cent.?
6. 8 per cent.? $\frac{3}{4}$ per cent.?
7. $\frac{5}{8}$ per cent.? $2\frac{5}{8}$ per cent.?
8. $\frac{1}{5}$ per cent.? $9\frac{5}{8}$ per cent.?
9. 16·2 per cent.? ·98 per cent.?
10. 147·2 per cent.? 26·12 per cent.?

What rate per cent. is equivalent to—

11. ·07 per unit? ·61 per unit?
12. 1·47 per unit? ·056 per unit?
13. ·8725 per unit? 2·2 per unit?
14. 1·11 per unit? 1·107 per unit?
15. 1·06 per unit? ·007 per unit?
16. 675 per unit? ·035 per unit?
17. ·014 per unit? ·016 per unit?
18. ·0095 per unit? 1·217 per unit?
19. ·00125 per unit? ·135 per unit?
20. ·0005 per unit? ·2775 per unit?

---

3. To find the percentage on any given number:—

RULE.

Multiply the given number by the rate per unit expressed decimally.

Example 1.—What is 16 per cent. of $674?

OPERATION.

16 per cent. = ·16 per unit.
$674 × ·16 = $107·84. *Ans.*

Example 2.—What is 7 per cent. of 8473 acres of and?

OPERATION.

8473 × ·07 = 593·11 acres = 593 a. 0 r. 17 per. 18 yds. 1 ft. $50\frac{2}{5}$ in.

Example 3.—What is 11 per cent. of 947 bushels of apples?

OPERATION.

947 × ·11 = 104·17 bush. *Ans.*

Example 4.—How much is 23 per cent. of $6147·80?

OPERATION.

23 per cent. = ·23 per unit.
$6147·80 × ·23 = $1413·994. *Ans.*

Exercise 62.

1. How much is 27 per cent. of $6090·80
2. What is 87½ per cent. of $1234?
3. What is 6¼ per cent. of $89·40?
4. How much is 17½ per cent. of $2998·40
5. What is 8¼ per cent. of 204 a. 2 r. 14 per.
6. How much is ·7 per cent. of 29 bush. 2 pks.?
7. What is ·72 per cent. of 429 lbs. 11 oz. 6 dwt.?
8. What is 15 per cent. of 227 weeks 4 days 11 hours
9. What is 6 per cent. of £93 14s. 7½d.?
10. What is 20 per cent. of $2947·40?
11. From 16 per cent. of $294 take 29 per cent. of $39·17.
12. Add together 7 per cent. of $94·80, 11 per cent. of $1129, and 17½ per cent. of $1296·42.
13. A person purchases a house for $7429 upon the following agreement:—He is to pay 15 per cent. of the purchase money down, 17 per cent. in 6 months, 29 per cent. in 15 months, 9 per cent. in 20 months, and the balance at the expiration of two years; what are his several payments, upon the supposition that he pays no interest?
14. A farmer works 227 acres of land, which he crops as follows:—20 per cent. in wheat, 18 per cent. in grass, 17 per cent. in peas, 19 per cent. in oats, and 8 per cent. in root crops, the rest being fallow; what number of acres does he sow to each crop and how much is in fallow?
15. A regiment went into the field 1147 strong, and after the battle it was found that 23 per cent. had been killed or wounded, and 7 per cent. taken prisoners; what was the number killed or wounded, and what the number taken prisoners?

COMMISSION AND BROKERAGE.

4. Commission is the percentage charged by agents or commission merchants for their services in buying or selling goods, collecting accounts, &c.

5. Brokerage is the percentage charged by money-dealers, called *brokers*, for negotiating notes, mortgages, bills of exchange, &c., or for buying or selling stock.

6. To compute commission or brokerage:—

RULE.

Multiply the given amount by the rate per unit expressed decimally.

Example 1.—What is the commission on $749·40 at 18 per cent.?

OPERATION.

18 per cent. = ·18 per unit.
$749·40 × ·18 = $134·892. *Ans.*

Example 2.—What is the commission on $198·37 at 22¾ per cent.?

OPERATION.

22¾ per cent. = ·2275 per unit.
$198·37 × ·2275 = $45·129175. *Ans.*

Example 3.—A broker purchases stock to the amount of $9867·30; what is his brokerage at the rate of 2⅛ per cent.?

OPERATION.

2⅛ per cent. = ·02125 per unit.
$9867·30 × ·02125 = $209·680125. *Ans.*

Exercise 63.

1. What is the commission on $79·80 at 4¼ per cent.?
2. What is the commission on $916·80 at 7⅝ per cent.?

3. What is the brokerage on $10800 at 1¼ per cent.?
4. What is the brokerage on $8877·66 at 3½ per cent.?
5. What is the brokerage on $678·90 at 5 per cent.?
6. What is the commission on $6719·50 at 8¾ per cent.?
7. What is the commission on $47·80 at 25 per cent.?
8. To what does the brokerage on $7654·32 amount at 4½ per cent.?
9. To what does the commission on $234·56 amount at 28 per cent.?
10. What is the commission on $555·55 at 18¾ per cent.?
11. An agent sells 617 bushels of wheat at $1·70 per bushel; what is his commission at 12½ per cent.?
12. A commission merchant sells goods to the amount of $1122·30; what is his commission at 33⅓ per cent.?
13. A broker purchases stock to the amount of $8765·40; what is his brokerage at 3¼ per cent.?
14. An agent purchases silks to the amount of $7800; what is his commission at 7 per cent.?
15. An agent collects debts to the amount of $907·80; what is his commission at 15 per cent.?
16. A commission merchant sells 7400 barrels of flour at $7·87½ per barrel; to what does his commission amount at 8½ per cent.?
17. An agent sells a farm for $7450; what is his commission at 2½ per cent.?
18. A broker negotiates a mortgage for the sum of $1140; what is his brokerage at 3⅜ per cent.?

INSURANCE.

7. Insurance is a written agreement by which an individual or an incorporated company becomes bound, in consideration of a certain sum paid in advance, to exempt the owners of certain kinds of property, as houses, household furniture, merchandise, ships, &c., from loss by fire, shipwreck, or other calamity.

8. The *Written Instrument*, or contract between the parties, is called a *Policy of Insurance.*

9. The sum paid for the insurance is called the *Premium*, and is usually a certain per cent. on the sum for which the property is insured.

10. Houses, merchandise, furniture, &c., are usually insured against risk of fire for the year, or other specified time.

NOTE.—The rate of insurance on dwelling-houses, stores, goods, household furniture, &c., varies from ¼ to 2 per cent. per annum on the sum insured, according to the character and position of the tenement. Vessels are insured for the voyage or the year.

11. The premium to be paid on a policy of insurance is computed by the following

RULE.

Multiply the sum to be insured by the rate per unit expressed decimally.

EXAMPLE.—If I insure my house and barn for $7480, what premium must I pay at 1¾ per cent. ?

OPERATION.

1¾ per cent. = ·0175 per unit, i. e. 1¾ cents is the charge for insurance on each $.

Then $7480 × ·0175 = $130·90. *Ans.*

EXERCISE 64.

Compute the insurance on—

1. $789·46 at 2¼ per cent.
2. $8167·50 at 2⅓ per cent.
3. $8900 at 3⅛ per cent.
4. $8740 at ⅞ per cent.
5. $1888 at ⅜ per cent.
6. $11247·60 at 1¼ per cent.
7. I wish to insure my house and furniture to the amount of $4780 ; what premium must I pay at 1⅜ per cent. ?
8. What must I pay for insuring a cargo of wheat worth $27490 from Toronto to Liverpool at 2⅜ per cent. ?
9. What premium of insurance, at 2⅛ per cent., must I pay on property valued at $8790 ?
10. What must I pay for insuring my house and barns to the amount of $17496·50 at ⅝ per cent. ?

BUYING AND SELLING STOCKS.

12. Stock is a term used to denote the *Capital* of moneyed institutions, as Banks, Railroad Companies, Gas Companies, Insurance Companies, Manufactories, &c.

13. Stock is usually divided into portions of $100 or £100 each, called *shares*, and the different individuals owning these are called *shareholders* or *stockholders*.

14. The *nominal* or *par* value of a share is its original cost or valuation.

15. The market or real value of a share is the sum for which it can be sold.

16. The rise and fall in the value of stock is reckoned at a certain per cent. on its *nominal* or *par* value.

17. When stocks sell for their original cost or valuation, they are said to be *at par;* when they sell for more than their original valuation, they are said to be *at a premium* or *advance*, or *above par;* when they do not bring their original cost or valuation, they are said to be *at a discount*, or *below par*.

NOTE.—*Par* is a Latin word, and means *equal* or *a state of equality*. Stock is *at par* when a hundred-dollar share sells for $100; it is *above par* when it brings more than $100, and *below par* when it will not bring as much as $100.

MENTAL EXERCISE.

1. When stock is selling at a premium of 17 per cent., what is $1 stock worth in money?

$100 stock = $117 money, therefore $1 stock = $1·17 money. *Ans.*

2. When stock is selling at a discount of 9 per cent., what is the worth of $1 stock?

$100 stock = $91 money, therefore $1 stock = $0·91 money. *Ans.*

3. When stock is 4½ per cent. above par, what is the value of $1 stock?

$100 stock = $104·50 money, therefore $1 stock = $1·045 money. *Ans.*

4. When stock is 9¾ per cent. *below par*, how much must be paid for $1 stock?

   $100 stock = $90·25 money, therefore $1 stock = $0·9025 money. *Ans.*

5. When stock is 7¼ per cent. *above par*, what is $1 stock worth?
6. When stock is 10 per cent. *below par*, what is $1 stock worth?
7. When stock is 8¼ per cent. *above par*, what is $1 stock worth?
8. When stock is *at par*, what is $1 stock worth?
9. When stock is at a premium of 34⅜ per cent., what is the worth of $1 stock?
10. When stock is at a discount of 2 per cent., what is the worth of $1 stock?

---

18. To find what sum must be paid for a given amount of stock:—

RULE.

Find the worth of $1 *stock and multiply it by the amount of stock.*

19. To find what amount of stock a given sum will purchase:—

RULE.

Divide the given sum by the worth of $1 *stock.*

Example 1.—What is the worth of $749·80 stock when it is selling at a premium of 8¾ per cent.?

OPERATION.

$100 stock = $108·75 money, therefore $1 stock is worth $1·0875.

Then $1·0875 × 749·80 = $815·4075. *Ans.*

Example 2.—What amount of stock at 7½ per cent. discount can be purchased for $1200?

OPERATION.

Here $100 stock = $92·50 money, and hence $1 stock is worth $0·925.

Then $1200 ÷ $0·925 = $1297·297. *Ans.*

Exercise 65.

1. How much stock can be purchased for $793 when it is selling at a premium of 17¼ per cent.?
2. What is the value of $9476 stock at 9¼ per cent. discount?
3. If I own 9 shares of stock in the Metropolitan Water Works, the par value of each share being $125, and sell out when the stock is at a premium of 8¾ per cent., what do I receive for my 9 shares?
4. When Upper Canada Bank stock is selling at a premium of 3¼ per cent., what must I pay for 17 shares, the par value being $111·216 per share?
5. When the stock of the Ontario and Huron Railway is 12 per cent. *below par*, how much should I pay for $6470 stock?
6. When the stock of the Hamilton Gas Works is selling at a premium of 6¼ per cent. I wish to invest $2000 in it; what amount of stock do I receive?
7. When stock is 27 per cent. above par, what amount can be purchased for $7000?
8. When stock is 8 per cent. below par, what is the value of $6140 stock?
9. When Grand Trunk Railway stock is selling at 1¼ per cent. premium, what must I pay for 27 shares, the par value being $25 per share?
10. When Montreal Bank stock is selling at a premium of 13¾ per cent., how much should I get for $11120?

SECTION VIII.

SIMPLE INTEREST, COMPOUND INTEREST, DISCOUNT, PARTNERSHIP.

1. Interest is the sum allowed for the use of money, and is usually reckoned at a certain rate per cent. per annum.

2. The sum lent is called the Principal.

3. The sum paid for the use of each hundred dollars is called the Rate Per Cent.

4. The whole sum received for the use of the principal is called the Interest.

5. The Amount is the sum obtained by adding together the principal and the interest.

Thus, if I lend $200 for a year on the agreement that I am to receive interest at the rate of 7 per cent., (*per annum* understood,) at the end of the year I receive back the $200, and in addition $14 for interest. Here,

$200·00 is the principal.
7·00 is the rate per cent.
0·07 is the rate per unit.
14·00 is the interest.
214·00 is the amount = principal + interest.

6. Interest is either Simple or Compound.

SIMPLE INTEREST.

7. Money is lent at Simple Interest when the interest is not added to the principal so as to bear interest.

8. Simple Interest is computed according to the following general

RULE.

Multiply the given principal by the rate per unit expressed decimally, and the product by the time The result will be the interest.

NOTE.—If the time consists of years, months, and days, we may express it decimally and then use it as multiplier ; or we may multiply by the years and take aliquot parts for the months and days.

EXAMPLE 1.—What is the interest on $759·80 at 7 per cent. for 1 year?

OPERATION.

$759·80 × ·07 = $53·186. *Ans.*

EXAMPLE 2.—What is the interest on $777·40 for 7 years at 6¼ per cent. per annum?

OPERATION.

$777·40 × ·0625 × 7 = $340·1125. *Ans.*

Example 3.—What is the interest of $6677 for 8 years 8 months 20 days at 5½ per cent.?

OPERATION.

		$6677	= principal.		
		·055	= rate per unit.		
		33385			
		33385			
6 mo.	½	$367·235	= interest for 1 year.		
		8			
		2937·880	=	"	8 years.
2 mo.	⅓	183·6175	=	"	6 months.
15 days	¼	61·20583	=	"	2 months.
5 days	⅓	15·30145	=	"	15 days.
		5·10048	=	"	5 days.
		$3203·10526	=	"	8 yrs. 8 mo [illegible] days.

Exercise 66.

Find the interest of—

1. $974 for 1 year at 11 per cent.
2. $1678·90 for 7 years at 9 per cent.
3. $142·70 for 16 years at 8 per cent.
4. $80·80 for 22 years at 7 per cent.
5. $67·49 for 6 years at 2¼ per cent.
6. $208·60 for 11 years at 3⅜ per cent.
7. $800 for 6 years 5 months 18 days at 8 per cent.
8. $7400 for 9 years 11 months 24 days at 6¼ per cent.
9. $9680·80 for 14 years 4 months at 3 per cent.
10. $476·76 for 10 years 8 months at 5¾ per cent.
11. $8900 for 6 years 7 months 28 days at 11¼ per cent.
12. $8160 for 9 years 15 days at 7½ per cent.
13. $412·90 for 6 years at 4⅝ per cent.
14. $127·40 for 3 years 3 months 3 days at 12¼ per cent.
15. $80·63 for 4·78 years at 2·97 per cent.
16. $100·70 for 11·113 years at 13·47 per cent.

---

9. Since the legal rate of interest in Canada is 6 per cent. when not otherwise specified by direct agreement,

it is important to have some simple rule by which interest at 6 per cent. can be computed.

10. To find the interest of $1 for any number of months at 6 per cent. :—

RULE.

Divide the number of months by 2 and call the quotient cents.

Example 1.—What is the interest of $1 for 8 months at 6 per cent.?

8 ÷ 2 = 4 cents. *Ans.*

Example 2.—What is the interest of $1 for 7 years 3 months at 6 per cent.?

7 years 3 months = 87 months, and 87 ÷ 2 = 43½ cents = $0·435. *Ans.*

Example 3.—What is the interest of $1 for 11 years 7 months at 6 per cent.?

11 years 7 months = 139 months, and 139 ÷ 2 = 69½ cents = $0·695. *Ans.*

11. To find the interest of $1 for any number of days at 6 per cent. :—

RULE.

Divide the number of days by 6 and call the result mills or tenths of a cent.

Example 1.—What is the interest of $1 for 18 days at 6 per cent.?

18 ÷ 6 = 3 mills = $0·003. *Ans.*

Example 2.—What is the interest of $1 for 26 days at 6 per cent.?

26 ÷ 6 = 4⅓ mills = $0·0043. *Ans.*

Example 3.—What is the interest of \$1 for 7 years 4 months 27 days at 6 per cent.?

7 years 4 months = 88 months, and 88 ÷ 2 = 44 cents = \$0·44 = interest for 7 years 4 months.

27 ÷ 6 = $4\frac{1}{2}$ mills = \$0·0045 = interest for 27 days.
Then \$0·4445 = interest for 7 years 4 months 27 days.

Exercise 67.

What is the interest of \$1 at 6 per cent. per annum for:

1. 8 mos.? 7 mos.? 11 mos.?
2. 2 years 9 months?
3. 16 years 4 months?
4. 5 years 11 months?
5. 11 years 1 month?
6. 10 years 10 months?
7. 4 years 5 months?
8. 6 years 3 months 12 days?
9. 3 years 3 months 3 days?
10. 4 years 7 months 10 days?
11. 1 year 9 months 25 days?
12. 2 years 7 months 17 days?

12. To find the interest of any sum of money for any time at 6 per cent. per annum:—

RULE.

Find by the last two rules the interest of \$1 for the given time and multiply it by the given principal.

Example 1.—What is the interest of \$67 for 2 years 3 months 12 days at 6 per cent.?

OPERATION.

Interest of \$1 for 2 years 3 months 12 days = \$0·137.
Then \$0·137 × 67 = \$9·179. *Ans.*

Example 2.—What is the interest of \$714·71 for 3 years 7 months 11 days at 6 per cent.?

OPERATION.

Interest of \$1 for 3 years 7 months 11 days = \$0·216$\frac{5}{6}$.
Then \$714·71 × 0·216$\frac{5}{6}$ = \$154·97295$\frac{1}{6}$. *Ans.*

Note.—When the number of days is not exactly divisible by 6, the interest for the days had better be written as mills and a *fraction* of a mill, and then the interest of \$1 for the given time, thus expressed, used for multiplier, as in the last example.

EXERCISE 68.

Find the interest at 6 per cent. per annum of:

1. $1904 for 7 years 9 months.
2. $274·80 for 4 years 11 months.
3. $671·90 for 2 years 2 months 12 days.
4. $213·27 for 3 years 3 months 3 days.
5. $49·73 for 4 years 4 months 4 days.
6. $619·80 for 5 years 5 months 5 days.
7. $27·60 for 6 years 6 months 6 days.
8. $47·32 for 7 years 7 months 7 days.
9. $222·22 for 8 years 8 months 8 days.
10. $345·67 for 9 years 9 months 9 days.
11. $789·23 for 10 years 10 months 10 days.
12. $809 for 11 years 11 months 11 days.
13. $207·40 for 3 years 24 days.
14. $98·20 for 1 year 28 days.
15. $76·42 for 2 years 7 months 15 days.
16. $9146·70 for 2 years 5 months 20 days.

COMPOUND INTEREST.

13. Money is lent at Compound Interest when the interest, as it falls due from time to time, is added to the principal; the sum thus obtained constituting a *new* principal for the ensuing year, half-year, quarter, &c., as the case may be.

14. To compute the Compound Interest on any sum of money for a given number of payments:—

RULE.

Find the interest on the given principal for one period, *i. e.*, ONE YEAR, HALF YEAR, *or* QUARTER, *as the case may be, and add it to the principal.*

Then find the interest on this amount for the NEXT PERIOD *and add it to the principal used for that period, as before.*

Proceed in this manner with each successive year or period of the proposed time.

Then the last result will be the amount of the given principal, at the given rate for the given time. Subtract the given principal from this, and the remainder will be the Compound Interest required.

EXAMPLE.—What is the compound interest of $700 for 2 years at 4 per cent. half-yearly?

OPERATION.

Here, since the interest is *half-yearly* there are four payments.

Interest of $700 at 4 per cent. = $28.

Then $700 + 28 = $728 = principal for 2d half year.

Interest of $728 at 4 per cent. = $29·12.

Then $728 + $29·12 = $757·12 = principal for 3d half year.

Interest of $757·12 at 4 per cent. = $30·2848.

Then $757·12 + $30·2848 = $787·4048 = principal for 4th half year.

Interest of $787·4048 at 4 per cent. = $31·496192.

Then $787·4048 + $31·496192 = $818·90 = amount at end of 4th half year.

From $818·90, the amount,
Take $700·00, the principal.
The remainder, $118·90, is the compound interest.

EXERCISE 69.

What is the compound interest of:

1. $1000 for 3 years at 7 per cent. per annum?
2. $800 for 4 years at 6 per cent. per annum?
3. $900 for 5 years at 6 per cent. per annum?
4. $600 for 2 years at 4 per cent. half yearly?
5. $250 for 2 years at 3½ per cent. half yearly?
6. $880 for 1½ years at 2 per cent. quarterly?

What are the amount and compound interest of:

7. $500 for 3 years at 7½ per cent. per annum?
8. $400 for 2 years at 4½ per cent. half yearly?
9. $714·90 for 2 years at 2½ per cent. quarterly?
10. $704·60 for 1½ years at 4½ per cent. half yearly?

DISCOUNT.

15. Discount is an allowance made for advancing the payment of a note, a mortgage, or other security, before it becomes due.

Thus, if I hold a note for any sum payable in two months and wish to obtain the money for it at once, I take it to my banker, who, after deducting his charge for advancing the money, pays me the balance.

16. The *present value* of a sum due at some future time, is what remains after deducting the discount.

17. The *true* discount on a note or other security is the interest on its present worth at the given rate per cent. and for the given time; but the *bank* discount (i. e. the discount as computed by bankers) is the interest on the sum named in the note, &c., at the given rate and for the given time.

18. To compute the *true* discount on a note or other security:—

RULE

Divide the sum for which the note, &c., is drawn by the amount of $1 for the given time, and at the given rate. The quotient will be the present worth. The discount is found by subtracting the present worth from the given sum.

Example 1.—What is the present worth of a note of $409, due 3 months hence, at 7 per cent. discount?

OPERATION.

7 per cent. per annum = $1\frac{3}{4}$ per cent. for 3 months.
$1\frac{3}{4}$ per cent. = $0·0175 = interest on $1 for 3 months at 7 per cent. per annum.

Hence amount of $1 at given rate and for given time = $1·0175.

Then $409 ÷ 1·0175 = $401·965. *Ans.*

Example 2.—What is the discount on a note for $794·63, due 27 days hence, discounting at 8 per cent.?

OPERATION.

Amount of $1 at 8 per cent. per annum for 27 days = $1·005917.

Then $794·63 ÷ $1·005917 = $789·955 = present worth.
And $794·63 ÷ $789·955 = $4·675 = discount.

Exercise 70.

What is the discount on :

1. A note of $740 drawn for 3 months, discount at 7 per cent.?
2. A note of $90 drawn for 2 months, discount at 9 per cent.?
3. A note of $250 drawn for 6 months, discount at 6 per cent.?
4. A note of $714·20 drawn for 11 months, discounting at 11 per cent.?
5. A note of $911·40 drawn for 5 months, discounting at 8 per cent.?
6. A note of $671·43 drawn for 4 months, discounting at 7 per cent.?
7. A bill of $947·60 drawn at 2 years, discounting at 4 per cent.?
8. A bill of $888·93 drawn at 1 year 4 months, discounting at 7 per cent.?
9. A bill of $7146·90 drawn at 47 days, discounting at 10 per cent.?
10. A bill of $710 drawn at 2 months, discounting at 7 per cent.?
11. A bill of $1100 drawn at 1½ months, discounting at 7 per cent.?
12. A bill of $6714·83 drawn at 2½ months, discounting at 6 per cent.?

BANK DISCOUNT.

19. As already remarked, the *bank* discount on any sum is the same as the interest on that sum, and hence to compute bank discount :—

RULE.

Add 3 days to the time which the note has to run before it becomes due, and calculate the interest for this time at the given rate per cent.

Note.—The 3 days added are the *days of grace*, or the 3 days which, by mercantile usage, are allowed to elapse, after a bill falls due, before it is payable. Bankers always add these 3 days to the time for which they compute discount.

Example 1.—What is the bank discount on a bill of $840, due 69 days hence, discounting at 7 per cent.?

OPERATION.

Interest of $840 at 7 per cent. for 1 year = $58·80.
69 + 3 = 72 days, and 72 days = $\frac{1}{5}$ of a year (360 days).
Hence bank discount = $\frac{1}{5}$ of $58·80 = $11·76. *Ans.*

Example 2.—What is the bank discount on a note of $471, due 3 months hence, discounting at 7 per cent.?

OPERATION.

Interest of $471 for 1 year at 7 per cent. = $32·97.
Time for which discount is charged = 3 months 3 days.

3 mos.	$\frac{1}{4}$	$32·97	Interest or bank discount for 1 year.		
3 days	$\frac{1}{30}$	8·2425	"	"	3 months.
		·2747	"	"	3 days.
		$8·5172	"	"	3 mos. 3 days.

Exercise 71.

Find the bank discount on:

1. A note of $700, due 42 days hence, discounting at 7 per cent.
2. A bill of $840, due 57 days hence, discounting at 8 per cent.
3. A bill of $790, due 4 months hence, discounting at 5 per cent.
4. A note of $614·30, due 2 months hence, discounting at 7 per cent.
5. A bill of $217·20, due 7 months hence, discounting at 9 per cent.
6. A note of $94·80, due 20 days hence, discounting at 10 per cent.

SIMPLE PARTNERSHIP.

80. Simple Partnership, called also Single Fellowship and Partnership Without Time, enables us to distribute the profits and losses of a firm or company equitably among its partners when they employ their shares for the same period of time.

21. The whole money employed in the business is called the *capital* or *stock.*

22. The profit or loss belonging to each member is calculated according to the following

RULE.

As the whole stock is to each man's share of the stock, so is the whole gain or loss to each man's share of the gain or loss.

Example.—A and B enter into partnership as grocers, with a capital of \$14000, of which A contributes \$8500 and B the remainder. They gain \$4740; what portion of this must each receive?

OPERATION.

Whole stock : A's stock :: Whole profit : A's profit.
That is, \$14000 : \$8500 :: \$4740 : A's profit, which is equal

$$\text{to } \frac{\$8500 \times 4740}{14000} = \$2877{\cdot}857.$$

Again, whole stock : B's stock :: whole profit : B's profit.
Or \$14000 : \$5500 :: \$4740 : B's profit, which is equal

$$\text{to } \frac{\$5500 \times 4740}{14000} = \$1862{\cdot}142.$$

Note.—After A's profit has been found by the rule, B's may be determined by subtracting A's share from the whole profit.

Exercise 72.

1. A, B, and C enter into business with a capital of \$7000, of which A contributes \$2700, B \$4200, and C the balance, and they gain \$1700; how must this be divided among them?
2. B and C together own a steamboat worth \$29000, of which B contributed \$17400 and C the balance. After paying all expenses for running her during the season, they find that they are losers to the amount of \$904·70; what portion of this loss must each sustain?
3. Three persons rent a pasture for the summer; the first puts in 21 cows, the second 17 cows, and the third 47 cows. The rent is \$307; what portion of this must each pay?
4. Three persons are to share \$7403 in the following manner, viz.: as often as A gets \$4 B gets \$7 and C gets \$9; what is the share of each?

5. A gentleman bequeathed $7500 to his three sons in proportion of 1¼; 2⅓, and 5¼; what was the share of each?

6. Two persons, A and B, enter into business with a capital of $8900, of which A contributes $4700 and B the rest. They gain $3200; what is the share of each?

7. Three persons have gained $9000, and agree to divide it in the following manner: as often as A takes $3 B takes $4·20 and C $7·49; what is the share of each?

8. A vessel worth $14900 is entirely lost, ¼ of it belonging to A, ⅜ to B, and the balance to C; what is the loss of each, $4250 being received as insurance?

9. Divide $4942 into three parts which shall be to one another as ½, ⅔, and ¾.

10. A merchant failing, owes A $7490 and B $2980, but to meet these demands he has only $7490; how much should each creditor receive?

COMPOUND PARTNERSHIP.

23. Compound Partnership, called also Double Fellowship or Partnership With Time, enables us to divide the profits or losses of any business fairly among the several partners when their stocks are invested for unequal terms.

24. Problems in Compound Partnership are solved according to the following

RULE.

Multiply each man's stock by the time he continues it in trade; then say, as the sum of the products is to each particular product, so is the whole gain or loss to each man's share of the gain or loss.

Example.—Three persons enter into partnership; A puts in $790 for 8 months, B $1145 for 7 months, and C $966 for 10 months. They gain $880; what portion of this should each receive?

OPERATION.

$$\begin{array}{rcl} \$\,790 \times \;\;8 = & \$6320 \\ 1145 \times \;\;7 = & 8015 \\ 966 \times 10 = & \underline{\;\;9660} \\ \text{Sum} = & \$23995 \end{array}$$

\$23995 : \$6320 :: \$880 : A's share, which is $\frac{6320 \times 880}{23995}$ = \$231·7816.

\$23995 : \$8015 :: \$880 : B's share, which is $\frac{8015 \times 880}{23995}$ = \$293·9445.

\$23995 : \$9660 :: \$880 : C's share, which is $\frac{9660 \times 880}{23995}$ = \$354·2738.

Note.—When two shares have been found, the third may be found by subtracting their sum from the whole profit or loss. So when there are four partners, and three shares have been found by the rule, the fourth may be obtained by subtracting the sum of these three from the whole profit or loss, &c.

Exercise 73

1. Two persons contract to make a road for \$4600. A furnishes 40 laborers for 37 days and B 30 laborers for 52 days; what part of the \$4600 should each receive?

2. Three persons enter into business as hardware merchants. A puts in \$2000 for 7 months, B \$1800 for 11 months, and C \$1600 for 12 months; if they gain \$2400, what is the share of each?

3. Two men undertake to drain a field for the sum of \$2400. A furnishes 10 men for 34 days and B 15 men for 36 days, and they have to pay \$400 out of the \$2400 for material; how much of the remaining \$2000 should each receive?

4. Three persons rent a pasture for \$120. A puts in 27 cows for 4 months, B 20 cows for 5 months, and C 24 cows for 3½ months; what portion of the rent should each pay?

5. Four persons begin business with a capital of \$10000, of which \$2400 belong to A, \$3000 to B, \$2000 to C, and the balance to D. A allows his contribution to remain in 11 months, B 4 months, C 9 months, and D 12 months, and they gain \$2500; how should this be divided among them?

6. A, B, and C, in partnership, have made $950; what are their respective shares of this, supposing A's capital in the business to have been $700 for 8 months, B's $1000 for 11 months, and C's $600 for 1 year and 5 months?

7. A commenced business with a capital of $10000, and at the end of 7 months B enters into partnership with him and brings in a capital of $7000, at which time A withdraws $4000 of his stock. At the end of the year they have gained $3000; what portion of this should each receive?

8. A builds a steamer which costs him $35000; 2 months after she is launched, B buys from him $11000 worth of stock in it, and in 3 months more C purchases $4000 worth of stock also from A. They run the boat for 7 months and gain during that time $9700; what portion of this belongs to each?

SECTION IX.

PROFIT AND LOSS, BARTER, EXCHANGE OF CURRENCIES, AND ANALYSIS.

PROFIT AND LOSS.

1. Profit and Loss enables us to ascertain how much we gain or lose on any mercantile transaction, and also how much we must increase or diminish the price of our goods in order to make a certain gain or loss per cent.

CASE I.

2. *To find the total gain or loss* on a certain quantity of goods when the prime cost and selling price are given:—

RULE.

Find the difference between the buying and the selling price of a bushel, lb., yard, &c.

Multiply the gain or loss per bushel, lb., yard, &c., by the number of bushels, lbs., or yards, and the result will be the whole gain or loss.

Example 1.—What do I gain if I buy 704 barrels of flour at $4·25 and sell it again at $4·93 per barrel?

OPERATION.

From $4·93 the selling price,
Take $4·25 the buying price.

The remainder $0·68 is the gain per barrel.
Then $0·68 × 704 = $478·72, the whole gain.

Example 2.—If I buy 1640 bushels of oats at 37½ cents per bushel and sell them at 35¼ cents, what do I lose on the transaction?

OPERATION.

From 37½ cents, the buying price,
Take 35¼ cents, the selling price.

The remainder 2¼ cents = the loss per bushel.
Then 2¼ cents × 1640 = $36·90. *Ans.*

Exercise 74.

1. If I buy 209 yards of flannel at 62½ cents per yard, and sell it again at 70 cents, what do I gain on the transaction?
2. If I purchase 8900 bushels of wheat at $1·29 and sell it again at $1·42, what do I gain on the transaction?
3. Suppose I sell 780 cedar posts at 12½ cents each, which I bought at 16 cents each, what do I lose on the transaction?
4. Bought 1142 thousand bricks at $4·92 per thousand and sold them at $5·47, what is my entire gain?
5. Bought 17 cwt. 2 qrs. 11 lbs. of butter at 18 cents per lb. and sold it at $23 per cwt.; what is my entire gain?
6. Bought 1143 lbs. of maple sugar at 11 cents per lb. and sold it at $13·50 per cwt., what is my entire gain?
7. If I purchase 63 tons of hay at $17·43 per ton and have to sell it at $12·94, what is my entire loss?
8. If I purchase 47 sheep at $3·37½ each and sell them at $4·17, what is my entire gain?

CASE II.

3. *To find at what price* an article must be sold, so as to gain or lose a certain percentage, the cost price being given:—

RULE.

Find the gain or loss per unit, and hence how much must be received for each dollar of the buying price.

Then multiply this by the whole cost price. The result will be the selling price.

Example 1.—Bought a quantity of wheat at $1·60 per bushel, and desire to sell so as to gain 14 per cent.; what must I charge per bushel?

OPERATION.

I want to gain $14 on $100, or 14 cents on $1.
Hence my selling price must be $1·14 × 1·60 = $1·824. *Ans.*

Example 2.—Bought a quantity of lumber for $7437·80; for what must I sell it so as to gain 16 per cent.?

OPERATION.

For every $100 I pay I want to get $116, therefore for every $1 paid away I require to receive $1·16.
Hence I must sell for $1·16 × 7437·80 = $8627·848. *Ans.*

Example 3.—Bought a quantity of flour for $7190 and agree to sell at a loss of 6 per cent.; what do I get for the whole?

OPERATION.

For every $100 I paid I agree to take $94, therefore for $1 paid I receive $0·94.
Therefore I get for the whole $0·94 × 7190 = $6758·60.

Exercise 75.

1. Bought a quantity of tea at $293; for what must I sell it in order to gain 10 per cent.?

2. Bought a quantity of leather for $890; for what must I sell it in order to gain 17 per cent.?

3. Bought 630 bushels of wheat at $1·23 per bushel, and agree to sell it at a loss of 8 per cent.; what do I receive for the whole?

4. Bought 950 saw logs at 44 cents each; for what must I sell the lot in order to gain 33 per cent.?

5. Bought 411 barrels of flour at $5·22 per barrel; for what must I sell the whole in order to gain 12½ per cent.?

6. Bought 512 dozen brooms at $2·80 per dozen, and agree to sell at a loss of 15 per cent.; what do I receive for the whole?

7. Bought 64980 dozen eggs at 7 cents per dozen, and sell so as to gain 24 per cent.; what do I receive for the whole lot of eggs?

8. Bought 908 tons of coal at $5·22 per ton, and agree to sell at a loss of 1½ per cent.; what do I get for the whole?

CASE III.

4. *To find the rate per cent. of profit or loss* when the buying price and the selling price are given:—

RULE.

Find the difference between the buying price and the selling price; this will be the whole gain or loss.

Then say, as the buying price is to 100 *so is the whole gain or loss to the gain or loss per cent.*

Example 1.—If I buy a house for $2700 and sell it for $3050, what is my gain per cent.?

OPERATION.

From $3050, the selling price,
Take $2700, the buying price.
The difference, $350, is the whole gain.
Then $2700 : $100 :: $350 : the gain on $100, i. e., the gain per cent.

Hence gain per cent. $= \frac{100 \times 350}{2700} = 12\frac{26}{27}$. *Ans.*

EXAMPLE 2.—Bought a quantity of wood for \$790 and sold it for \$750; what was my loss per cent.?

OPERATION.

$\$790 - \$750 = \$40 =$ whole loss.

Then $\$790 : \$100 : : 40 : \dfrac{100 \times 40}{790} = 5\tfrac{5}{79}$ per cent. *Ans.*

EXERCISE 76.

1. Bought hay at \$24 per ton and sold it at \$26·25; what was my gain per cent.?
2. Bought 279 barrels of pork at \$17·80 per barrel and sold the whole for \$5570; what was my gain per cent.?
3. Bought 212 barrels of apples at \$2·30 per barrel and sold the whole for \$600; what was my gain per cent.?
4. Bought 93 barrels of cider at \$7·40 per barrel and sold the whole for \$651, what was my loss per cent.?
5. Bought 205 horses at an average cost of \$93·40 each and sold the whole for \$20987; what was my gain per cent.?
6. Bought a farm for \$7400 and sold it for \$6250; what was my loss per cent.?

CASE IV.

5. *To find the cost price* when the selling price and the gain or loss per cent. are given:—

RULE.

As \$100 + *gain per cent.* (or \$100 — *loss per cent.*) *is to* \$100 *so is the selling price to the cost price.*

EXAMPLE 1.—What sum did I pay for a carriage which I sold for \$317, gaining 11 per cent.?

OPERATION.

$\$100 + \$11 = \$111.$

Then $\$111 : \$100 : : \$317 : \dfrac{100 \times 317}{111} = \$285{\cdot}585.$ *Ans.*

Example 2.—Sold a quantity of butter for $2147, losing thereby 7 per cent. on the transaction; what did it cost me?

OPERATION.

$$\$100 - \$7 = \$93.$$

$$\text{Then } \$93 : \$100 :: \$2147 : \frac{100 \times 2147}{93} = \$2308{\cdot}602. \textit{ Ans.}$$

Exercise 77.

1. What did I pay per bushel for wheat which I sold for $1·70 at a gain of 18 per cent.?
2. What did I pay for a quantity of shingles which I sold for $324, gaining 29 per cent.?
3. Sold 356 bushels of clover seed for $1780, losing thereby 11 per cent.; what did it cost me per bushel?
4. What did I pay for butter upon which I lost 14 per cent. selling it at $8\frac{6}{10}$ cents per lb.?
5. Sold a grist-mill for $9490 and gained 43 per cent. on the transaction; what did I pay for it?
6. An agent sells 209 barrels of flour for me at $6·72 per barrel. Now this was 11 per cent. above the cost price, but I have to pay my agent 20 per cent. for commission; what does the flour stand me per barrel?
7. Sold a horse for $145 and gained 9 per cent. on the transaction; what did the horse cost me?
8. What did I pay for two-inch draining tiles which I sold for $12 per 1000, gaining 31 per cent.?

BARTER.

6. Barter enables two parties to make an exchange of goods at prices agreed upon so that neither shall suffer loss.

7. Questions in barter are solved by the following

RULE.

Find the value of the commodity whose price and quantity are given.

Divide this by the price of the other commodity and the quotient will be the quantity; or divide by the quantity and the quotient will be the price.

Example 1.—How much tea at \$0·85 per lb. must a farmer receive for 211 bushels of turnips at 23 cents per bushel?

OPERATION.

211 bushels of turnips at 23 cents = \$48·53.
Then \$48·53 ÷ \$0·85 = 57·094 lbs. = 57 lbs. $1\frac{1}{2}$ oz.

Example 2.—A has 307 yards of linen at 63 cents per yard and barters it with B for 20 cwt. of sugar· what does B get per lb. for his sugar?

OPERATION.

307 yards at 63 cents = \$193·41.
20 cwt. of sugar = 2000 lbs.
Then \$193·41 ÷ 2000 = \$0·0967 = $9\frac{67}{100}$ cents. *Ans.*

EXERCISE 78.

1. A has 207 lbs. of sugar at 11 cents per lb., which he barters with B for $34\frac{1}{4}$ lbs. of tea; what does the tea stand B per lb.?
2. A farmer delivers 293 dozen eggs at 10 cents per dozen and takes in payment 47 lbs. of raisins at 18 cents per lb., 9 lbs. of loaf-sugar at 14 cents per lb., 23 lbs. of rice at 6 cents per lb., and the balance in nails at 5 cents per lb.; how many lbs. of nails does he receive?
3. A grocer had 91 lbs. of figs at 27 cents per lb., and bartered them with a dry-goods merchant for drugget at 43 cents per yard; how much drugget did he receive?
4. A farmer has 0 cows which he values at \$33·70 each and barters them for 84 sheep; how much does he give for each sheep?
5. B has 68 yards of silk worth \$1·73 per yard and barters it with C for 25 yards of broadcloth and \$14·20 in money; what does the cloth cost him per yard?
6. A farmer has 409 lbs. of cheese which he barters with a neighbor for 607 lbs. of pork at \$4·90 per cwt.; how much does he receive per lb. for his cheese?

7. A farmer carries to a grist-mill 420 bushels of wheat, worth $1·39 per bushel, and receives in payment $207·50 and 11983¼ lbs. of flour; how much does the miller charge per cwt. for his flour.

8. B has 423 lbs. of sugar which is worth 11 cents per lb. and he barters it with C for golden syrup worth 23 cents per quart; how much syrup does he receive?

EXCHANGE OF CURRENCIES.

8. Table of Currencies in Canada and the United States.

In Canada, Nova Scotia, New Brunswick, &c.	$1 = 5s.	or £$\frac{1}{4}$.
In New York, North Carolina, Ohio, and Michigan	$1 = 8s.	or £$\frac{2}{5}$.
In New England, Virginia, Kentucky, Tennessee, Indiana, Illinois, Mississippi, Missouri	$1 = 6s.	or £$\frac{3}{10}$.
In Pennsylvania, New Jersey, Delaware, and Maryland	$1 = 7s. 6d.	or £$\frac{3}{8}$.
In Georgia and South Carolina	$1 = 4s. 8d.	or £$\frac{7}{30}$.

NOTE.—These pounds, shillings, and pence are not *coins*, but are merely the denominations employed in keeping accounts. The remaining States use the Federal money exclusively.

---

9. To reduce dollars and cents to old Canadian Currency, or to any State Currency:—

RULE.

Multiply the given sum by that fraction of £1 which expresses the value of $1; the product will be pounds and decimals of a pound.

Then reduce the decimals to shillings, pence, and farthings.

EXAMPLE 1.—Reduce $207·43 to old Canadian Currency.

OPERATION.

$1 = £$\frac{1}{4}$. Then $207·43 × $\frac{1}{4}$ = £51·8575.
And £51·8575 = £51 17s. 1$\frac{4}{5}$d. *Ans.*

Example 2.—Reduce \$294·80 to Kentucky currency.

OPERATION.

$\$1 = £\frac{3}{10}$. Then $\$294·80 \times \frac{3}{10} = £88·44$.
$= £88$ 8s. $9\frac{3}{5}$d. *Ans.*

---

10. To reduce old Canadian Currency or any State Currency to dollars and cents:—

RULE.

Express the given sum decimally and divide it by the value of \$1 expressed as a fraction of a pound. The quotient will be dollars and cents, &c.

Example 1.—Reduce £227 8s. $4\frac{3}{4}$d. old Canadian Currency to dollars and cents.

OPERATION.

£227 8s. $4\frac{3}{4}$d. $= £227·41979$.
Then $227·41979 \div \frac{1}{4} = 227·41979 \times 4 = \$909·679$. *Ans.*

Example 2.—Reduce £411 6s. $7\frac{1}{2}$d. Michigan Currency to dollars and cents.

OPERATION.

£411 6s. $7\frac{1}{2}$d. $= £411·33125$.
Then $411·33125 \div \frac{2}{5} = 411·33125 \times \frac{5}{2} = \$1028·328$. *Ans.*

---

11. To reduce dollars and cents to sterling money:—

RULE.

Divide the given sum by the value of £1 sterling (\$4·867). *The result will be pounds sterling and decimals of a pound.*

Then reduce the decimal to shillings and pence.

Example 1.—Reduce \$1479·83 to sterling money.

OPERATION.

$\$1479·83 \div 4·867 = £304·05383 = £304$ 1s. $0\frac{9}{10}$d.

12. To reduce sterling money to dollars and cents:—

RULE.

Express the given sum decimally and multiply it by the legal value of £1 *sterling* ($4·867).

Example 2.—Reduce £29 4s. 7d. sterling to dollars and cents.

OPERATION.

£29 4s. 7d. =£29·229166.

Then £29·229166 × 4·867 = $142·25835. *Ans.*

Exercise 79.

1. Reduce $714·93 to old Canadian currency.
2. Reduce $914·90 to North Carolina currency.
3. Reduce $611·20 to New England currency.
4. Reduce $43·92 to Ohio currency.
5. Reduce £293 17s. 4d. old Canadian currency to dollars and cents.
6. Reduce £294 11s. 11¼d. New Jersey currency to dollars and cents.
7. Reduce £247 2s. 5¼d. South Carolina currency to dollars and cents.
8. Reduce £89 11s. 10¼d. New Brunswick currency to dollars and cents.
9. Reduce $994·70 to sterling money.
10. Reduce $890·93 to sterling money.
11. Reduce $1020·11 to sterling money.
12. Reduce $80·74 to sterling money.
13. Reduce £29 14s. 11¼d. sterling to dollars and cents.
14. Reduce £294 16s. 2¼d. sterling to dollars and cents.
15. Reduce £411 16s. 7d. sterling to dollars and cents.
16. Reduce £843 9s. 0d. sterling to dollars and cents.
17. Reduce £294 11s. 10d. Delaware currency to dollars and cents.
18. Reduce $2047·80 to sterling money.
19. Reduce $1291·10 to New York currency.
20. Reduce £470 19s. 8¼d. sterling to dollars and cents.

ANALYSIS.

13. Analysis in arithmetic is the process of solving problems independently of set rules, by tracing the relations of the given numbers, and reasoning *from the given number to unity* and *from unity to the required number.*

NOTE.—Many of the preceding operations are worked by the analytical method, as, for example, Percentage, Insurance, Commission, &c., but it has been thought advisable to illustrate, at this point, the mode of applying the method to a variety of other problems.

EXAMPLE 1.—If 7 bushels of peaches cost $28, how much will 12 bushels cost?

OPERATION.

If 7 bushels cost $28, 1 bushel will cost $\frac{1}{7}$ of $28, that is, $4. Now, if 1 bushel cost $4, 12 bushels will cost 12 times $4, that is, $48. *Ans.*

EXAMPLE 2.—$\frac{7}{11}$ of 88 are how many times 5?

OPERATION.

$\frac{1}{11}$ of 88 is 8, and therefore $\frac{7}{11}$ of 88 is $8 \times 7 = 56$. and 5 is contained in 56 $11\frac{1}{5}$ times. *Ans.*

EXAMPLE 3.—A person bought a horse and paid $72 cash, and this was $\frac{6}{11}$ of the price of it: what did it cost?

OPERATION.

If $72 is $\frac{6}{11}$ of the price, $\$72 \div 6 = \$12$, must be $\frac{1}{11}$ of the price, and therefore the price is $\$12 \times 11 = \$132$. *Ans.*

EXAMPLE 4.—If 28 men can do a piece of work in 42 days, in how many days can 21 men do it?

OPERATION.

If 28 men require 42 days, 1 man will require 28 times 42 days.

Then, if 1 man requires 28 times 42 days, 21 men will require the $\frac{1}{21}$ part of 28 times 42 days, that is, $\frac{28 \times 42}{21} = 56$ days. *Ans.*

EXAMPLE 5.—A can do a piece of work in 7 days which B can do in 5 days; in what time can they do it if they work together?

OPERATION.

If A can do the whole work in 7 days, he can do $\frac{1}{7}$ of the work in 1 day; and if B can do the whole work in 5 days, he can do $\frac{1}{5}$ of it in 1 day.

Then, since A does $\frac{1}{7}$ and B $\frac{1}{5}$ in one day, they will together do $\frac{1}{7} + \frac{1}{5}$, which is $\frac{12}{35}$ of the work, in 1 day; and to do the whole work, they will require as many times $\frac{12}{35}$ of a day as $\frac{12}{35}$ is contained times in 1.

Then the time required will be $1 \div \frac{12}{35} = 1 \times \frac{35}{12} = \frac{35}{12} = 2\frac{11}{12}$ days. *Ans.*

EXAMPLE 6.—A, B, and C can together do a piece of work in 30 days, A alone can do it in 75 days, and B working alone can do it in 80 days; in what time would C working alone do it?

OPERATION.

A, B, and C can together do it in 30 days, therefore in 1 day they can do $\frac{1}{30}$ of the work.

A working alone requires 75 days, therefore in 1 day he can do $\frac{1}{75}$ of the work; B working alone requires 80 days, therefore in 1 day he can do $\frac{1}{80}$ of the work. Hence A and B working together will do $\frac{1}{75} + \frac{1}{80} = \frac{31}{1200}$ of the work in 1 day, but A, B, and C do $\frac{1}{30}$ in 1 day.

Therefore C must do the difference between $\frac{1}{30}$ and $\frac{31}{1200}$, that is, $\frac{1}{30} - \frac{31}{1200} = \frac{9}{1200} = \frac{3}{400}$.

And, since C does $\frac{3}{400}$ in 1 day, to do the whole work he would require $1 \div \frac{3}{400} = 1 \times \frac{400}{3} = \frac{400}{3} = 133\frac{1}{3}$ days. *Ans.*

EXAMPLE 7.—A grocer mixes together 7 lbs. of sugar at 9 cents per lb., 4 lbs. at 12 cents, and 6 lbs. at 10 cents; what should he charge per lb. for the mixture?

OPERATION.

7 lbs. at 9 cents	will come to	63 cents.
4 lbs. at 12 "	"	48 "
6 lbs. at 10 "	"	60 "
17		171

Therefore the mixture contains 17 lbs. of sugar and is worth 171 cents; but if 17 lbs. be worth 171 cents, 1 lb. should be worth $\frac{1}{17}$ of 171, that is, $171 \div 17 = 10\frac{1}{17}$ cents. *Ans.*

EXAMPLE 8.—In a certain school $\frac{1}{3}$ of the scholars are at arithmetic, $\frac{2}{7}$ at writing, $\frac{1}{5}$ at geography, and the rest, which is 38, at play; how many scholars are there in the school?

OPERATION.

The sum of $\frac{1}{3}$, $\frac{2}{7}$, and $\frac{1}{5} = \frac{86}{105}$, therefore the number at play must be the whole minus $\frac{86}{105}$, that is, $\frac{105}{105} - \frac{86}{105} = \frac{19}{105}$.

But the number at play is 38, and hence 38 is $\frac{19}{105}$ of the whole school.

If 38 is $\frac{19}{105}$, $\frac{1}{105}$ is $\frac{1}{19}$ of 38, which is 2.

If 2 is $\frac{1}{105}$, $\frac{105}{105}$, i. e. the whole school, will be $2 \times 105 = 210$. *Ans.*

EXAMPLE 9.—Two persons start at the same time in opposite directions to walk from Toronto to Hamilton, a distance of 38 miles. A travels from Toronto at the rate of 5 miles per hour, and B from Hamilton at the rate of 4 miles per hour; when and where will A and B meet?

OPERATION.

It is evident they approach each other at the rate of $5 + 4 = 9$ miles per hour, and hence the *time* will be $38 \div 9 = 4\frac{2}{9}$ hours. Next, A travels $4\frac{2}{9}$ hours at the rate of 5 miles per hour, therefore they will meet $4\frac{2}{9} \times 5 = 21\frac{1}{9}$ miles from Toronto, or $4\frac{2}{9} \times 4 = 16\frac{8}{9}$ miles from Hamilton.

EXERCISE 80.

1. How many times 3 are $\frac{3}{11}$ of 77?
2. How many times 5 are $\frac{4}{7}$ of 49?
3. How many times 7 are $\frac{9}{10}$ of 130?
4. How many times 9 are $\frac{3}{5}$ of 70?
5. How many times 12 are $\frac{2}{3}$ of 54?
6. 72 is $\frac{6}{11}$ of how many times 5?
7. 121 is $\frac{11}{12}$ of how many times 10?
8. 48 is $\frac{6}{7}$ of how many times 7?
9. 78 is $\frac{6}{11}$ of how many times 11?
10. $\frac{3}{5}$ of 25 is $\frac{5}{7}$ of what number?
11. $\frac{5}{6}$ of 42 is $\frac{5}{11}$ of what number?
12. $\frac{2}{3}$ of 81 is $\frac{3}{10}$ of what number?
13. $\frac{4}{11}$ of 99 is $\frac{3}{4}$ of what number?
14. If 8 cows give 44 lbs. of butter per week, how much may be expected from 11 cows?
15. If $27 pay for 9 barrels of apples, what will 23 barrels of apples cost?
16. If 13 days' work cost $7·80, for how many days' work will $19·80 pay?
17. A can do a piece of work in 9 days which A and B working together can do in 4 days; in what time could B alone do it?
18. A can do a piece of work in 10 days which B could do in 7 days, and C in 12 days; if they all three work together at it, in what time can they finish it?
19. A, B, and C can together do a piece of work in 15 days. A alone can do it in 35 days and C alone in 42 days. In what time could B working alone finish it?
20. A person bought a cow and paid $10·5 cash, and this was $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{13}{8}$ of the price; how much did he give for the cow?

21. A person bought a farm and paid \$1000 cash, and this was $\frac{2}{3}$ of $\frac{9}{4}$ of $\frac{6}{25}$ of $\frac{10}{27}$ of the price; what did the farm cost him?

22. A farmer mixes 27 bushels of wheat worth \$1·40 per bushel with 11 bushels worth only \$1·10; what is the mixture worth per bushel?

23. A wine merchant mixes 15 gallons of wine worth \$4·80 per gallon with 12 gallons at \$3·70 and 10 gallons at \$2·90; what is the mixture worth per gallon?

24. Having counted my sheep I found that $\frac{1}{2}$ + $\frac{2}{3}$ + $\frac{1}{6}$ of them numbered 80; how many had I in my flock?

25. A post is $\frac{1}{4}$ of its length in the ground, $\frac{2}{3}$ in water, and 9 feet above the water; what is the length of the post?

26. A and B started at the same time to walk in opposite directions around an island 100 miles in circumference, A walking 3 and B 3½ miles per hour; when and where will they meet?

27. A and B start from the same point, and at the same time, and in the same direction, to walk round a circular island 60 miles in circumference, A travelling at the rate of 5 and B at the rate of 3½ miles per hour; how many miles will A travel before he overtakes B?

28. What number is it that the $\frac{1}{3}$ and $\frac{1}{4}$ and $\frac{1}{5}$ and $\frac{1}{6}$ of which make 104?

29. What number is that $\frac{3}{4}$ of which exceeds $\frac{8}{11}$ of it by 2?

30. A certain number is divided by 4 and from $\frac{5}{8}$ of the quotient 5 is subtracted; the remainder is then divided by 11, 7 is added to the quotient, and the sum multiplied by 3; now $\frac{1}{4}$ of $\frac{4}{5}$ of $\frac{7}{8}$ of this product is 11$\frac{1}{3}$; what was the original number?

SECTION X.

INVOLUTION, EXTRACTION OF SQUARE ROOT AND OF CUBE ROOT.

INVOLUTION.

1. A power of any number is the product obtained by multiplying that number by itself one or more times.

Thus $25 = 5 \times 5$ is a power of 5; $81 = 3 \times 3 \times 3 \times 3$ is a power of 3, &c.

2. The number which, being multiplied once or oftener by itself, produces the power, is called the *root* of that power.

Thus 5 is the root of 25, since $5 \times 5 = 25$; 3 is the root of 81, since $3 \times 3 \times 3 \times 3 = 81$.

3. The powers of a number are called the *first*, *second*, *third*, *fourth*, *fifth*, &c., according as the root is taken *once*, *twice*, *thrice*, *four times*, *five times*, &c., as factor.

Thus 81 is called the fourth power of 3, because 3 is taken 4 times as factor, in order to produce 81.

4. The second power of a number is also called its *square*, because a square surface, the length of one of whose sides is expressed by a given number, will have its area expressed by the second power of that number.

5. The third power of a number is also called its *cube*. because if the length of one side of a cube be expressed by a given number, the solid contents of the cube will be expressed by the third power of that number.

TABLE OF SQUARES AND CUBES.

Roots...	1	2	3	4	5	6	7	8	9
Squares.	1	4	9	16	25	36	49	64	81
Cubes...	1	8	27	64	125	216	343	512	729

6. The *index* or *exponent* of a power is a small figure written to the right, indicating how often the root has to be taken as factor in order to produce the given power.

Thus, $2 = 2 = 2 =$ First power of 2.
$2^2 = 2 \times 2 = 4 =$ Second power of 2.
$2^3 = 2 \times 2 \times 2 = 8 =$ Third power of 2.
$2^4 = 2 \times 2 \times 2 \times 2 = 16 =$ Fourth power of 2.
$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32 =$ Fifth power of 2.

7. The process of finding a power of a given number by multiplying it into itself is called involution.

---

8. To involve a number to any required power:—

RULE.

Take the given number as factor as often as indicated by the index of the required power and find the product of these factors.

NOTE.—*To involve fractions, we multiply both numerators and denominators as above; to involve mixed numbers, we reduce them to improper fractions and then proceed as above.*

EXAMPLE 1.—What is the 5th power of 4?

OPERATION.

$$4^5 = 4 \times 4 \times 4 \times 4 \times 4 = 1024. \quad Ans$$

EXAMPLE 2.—What is the 3d power of $2\frac{1}{5}$?

OPERATION.

$$(2\tfrac{1}{5})^3 = (\tfrac{11}{5})^3 = \tfrac{11}{5} \times \tfrac{11}{5} \times \tfrac{11}{5} = \frac{11 \times 11 \times 11}{5 \times 5 \times 5}$$
$$= \tfrac{1331}{125} = 10\tfrac{81}{125}. \quad Ans.$$

EXERCISE 81.

Find the value of—

1. The *square* of 17.
2. The *cube* of 23.
3. The *square* of 279.
4. The *cube* of 81.
5. The *fourth power* of 6.
6. The *fifth power* of 5.
7. The *sixth power* of 4.
8. The *seventh power* of 3.
9. The *eighth power* of 2.
10. The *ninth power* of 3.
11. The *third power* of 7.
12. The *fourth power* of 11.
13. The *sixth power* of 9.
14. The *fifth power* of $\frac{3}{5}$.
15. The *square* of 1225.
16. The *square* of 4837
17. The *cube* of $4\frac{1}{2}$.
18. The *cube* of 29.
19. The *fourth power* of $2\frac{3}{8}$.
20. The *tenth power* of 3.

EXTRACTION OF THE SQUARE ROOT.

9. To extract the square root of a number:—

RULE.

I. *Point off the given number into periods of two figures each, beginning at the decimal point.*

II. *Find the highest square contained in the left-hand period, and place its root to the right of the number, in the place occupied by the quotient in division.*

III. *Subtract the square of the digit put in the root from the left-hand period, and to the remainder bring down the next period, to the right, for a new dividend.*

IV. *Double the part of the root already found for a* TRIAL DIVISOR.

V. *Find how many times the trial divisor is contained in the dividend, exclusive of the right-hand digit, and place the figure thus obtained both in the root and also to the right of the trial divisor.*

VI. *Multiply the divisor thus completed by the digit last put in the root; subtract the product from the dividend, and to the remainder bring down the next period for a new dividend.*

VII. *Again, double the part of the root already found for a new* TRIAL DIVISOR; *proceed as in V. and VI., and continue the process until all the periods are brought down.*

NOTE.—To extract the square root of a fraction, extract the square root of the numerator and of the denominator separately, if they be complete squares; if not, reduce the fraction to its equivalent decimal and extract the square root by the rule. To extract the square root of a mixed number, reduce the fractional part to a decimal attached to the whole number and then extract the square root.

EXAMPLE 1.—What is the square root of 576?

OPERATION.

```
  ·
  576(24
  4
  ___
44)176
   176
```

Here we place a point between the 7 and the 5 and thus divide the number into two periods. Then the highest square in 5, the first period, is 4, the square root of which, 2, we place in the root. Next we subtract the 4 from 5 and bring down the next period, 76, which gives us 176 for the next dividend. Then we double the 2 in the root for a trial divisor and ask how often this 4 will go into 17 (the dividend exclusive of the right-hand figure); obviously 4 times; next we place this 4 both in the root and in the divisor, multiply the complete divisor thus formed by the 4 and subtract.

Example 2.—What is the square root of $3226\frac{4}{17}$?

OPERATION.

$3226\frac{4}{17} = 3226{\cdot}235294$

```
          3226·235294(56·7999, &c.
          25
          ---
   106)   726
          636
          ----
  1127)    90·23
           78·89
           -----
113·49)    11·3452
           10·2141
           -------
113·589)    1·131194
            1·022301
            --------
113·5989)    10889300
             10223901
             --------
               685399, &c.
```

Here, after reducing $\frac{4}{17}$ to a decimal and annexing it to the whole number, 3226, we mark off both ways from the decimal point into periods of two figures each. Then the highest square in 32, the left-hand period, is 25, the square root of which is 5, and we accordingly put 5 in the root; next we subtract the 25 from the 32 and bring down the next period, 26, which gives us 726 for a new dividend. Then we take twice 5 = 10, for a trial divisor; find how often 10 will go into 72, apparently 7 times; but when we try 7 we find that it is too great, and therefore we try 6, which we put both in the root and in the divisor. Next we multiply the 106 by 6 and subtract the product, 636, from 726, and to the remainder bring down the next period, which gives us 90·23 for a new dividend. Next we take twice 56 = 112, for a trial divisor, and find that it will go into 90·2 seven times, and we accordingly place 7 both in the divisor and in the root, multiply, subtract, and bring down, &c.

EXERCISE 82.

Find the square root of:

1. 1296.
2. 3969.
3. 15876.
4. 53361.
5. 142884.
6. 998001.
7. ·244036.
8. ·395641.
9. 756·25.
10. 11397·4849.
11. 98123·47891.
12. 6712914·23.
13. 918767.
14. $429\frac{1}{4}$.
15. $\frac{9}{16}$, $\frac{36}{81}$, $\frac{121}{169}$, $\frac{81}{400}$.
16. $\frac{7}{11}$, $\frac{9}{17}$.
17. $428\frac{3}{7}$.
18. $629\frac{4}{5}$.
19. $1127896\frac{1}{4}$.
20. 218798·1237.

EXTRACTION OF CUBE ROOT.

10. To extract the cube root of a number is to find a number which, taken *three times* as factor, will produce the given number:—

RULE.

I. *Point off the number into periods of three figures each, beginning at the decimal point.*

II. *Find the highest cube contained in the left-hand period and place its root to the right of the number, in the place occupied by the quotient in division.*

III. *Subtract the cube of the digit put in the root from the left-hand period, and to the remainder bring down the next period to the right for a new dividend.*

IV. *Multiply the square of the part of the root already found by 300 for a* TRIAL DIVISOR.

V. *Find how many times the trial divisor is contained in the dividend and put the figure thus obtained in the root.*

VI. *Complete the* TRIAL DIVISOR *by adding to it:*

1st. *The part of the root previously found* × *the last digit put in the root* × 30; *and*

2d. *The square of the last digit put in the root.*

VII. *Multiply the divisor thus completed by the digit last put in the root; subtract the product from the dividend and to the remainder bring down the next period for a new dividend.*

VIII. *Again multiply the square of the part of the root already found by* 300 *for a new* TRIAL DIVISOR; *find what digit to place next in the root, as in V.; complete the divisor by making the two additions to the trial divisor described in VI.; multiply, subtract, and bring down as directed in VII., and continue the process until all the periods are brought down.*

NOTE.—To extract the cube root of a fraction, reduce it to a decimal and then extract the cube root, unless both numerator and denominator be complete cubes, in which case extract the cube root of each. To extract the cube root of a mixed number, reduce the fractional part to a decimal, annex to the whole number, and then extract the cube root as directed above.

EXAMPLE.—What is the cube root of 429172932007?

OPERATION.

			429172932007(7543. *Ans.*
			343
1st trial divisor	$= 7^2 \times 300 =$	14700	86172 = 1st dividend.
1st increment	$= 7 \times 5 \times 30 =$	1050	
2d "	$= 5^2 =$	25	
1st complete divisor	=	15775	78875 = product of comp. div. by 5.
2d trial divisor	$= 75^2 \times 300 =$	1687500	7297932 = 2d dividend.
1st increment	$= 75 \times 4 \times 30 =$	9000	
2d "	$= 4^2 =$	16	
2d complete divisor	=	1696516	6786064 = product of comp. div. by 4.
3d trial divisor	$= 754^2 \times 300 =$	170554800	511868007 = 3d dividend.
1st increment	$= 754 \times 3 \times 30 =$	67860	
2d "	$= 3^2 =$	9	
3d complete divisor	=	170622669	511868007 = product of comp. div. by 3.

After pointing off, we find that the highest cube number contained in the left-hand period is 343, of which the cube root is 7. We therefore place 7 in the root and subtract 343 from the first period. This gives us a remainder of 86, to which we bring down the next period, 172, and thus obtain 86172 for a new dividend.

Next we take 7, the part of the root already found, square it and multiply the 49 thus obtained by 300. This gives the first trial divisor, 14700, which we find will go into the dividend 86172 (making due allowance for the increase of the divisor) 5 times.

Next we complete the divisor by adding to it

1st, 7 × 5 × 30 = 1050, and 2d, $5^2 = 25$, which gives us 15775 for a complete divisor. This we multiply by 5, the digit last put in the root, subtract the product 78875 from the 1st dividend, and to the remainder, 7297, bring down the next period, 932, &c., &c.

EXERCISE 83.

Extract the cube root of:

1. 32768.
2. 658503.
3. 13824.
4. 250047.
5. 970299.
6. 1953125.
7. 15813251.
8. 48228544.
9. 245314376.
10. 686·128968.
11. 991026·973.
12. ·915498611.
13. $\frac{8}{27}$, $\frac{125}{1728}$, $\frac{343}{729}$, $\frac{64}{125}$.
14. $\frac{11}{12}$, $\frac{9}{10}$.
15. $\cdot\dot{9}$, ·1, $\cdot\dot{1}$.
16. 427086·7143.
17. $816\frac{3}{8}$.
18. $917167\frac{4}{11}$.
19. $8111471\frac{9}{13}$.
20. $27\frac{1}{4}$.

EXERCISE 84.

Miscellaneous Problems.

1. Divide $7094·70 equally among 29 persons.
2. The difference of two numbers is 127 and the greater is 249, what is the smaller?
3. Reduce £294 6s. 4½d. to dollars and cents and divide the result by $\cdot\dot{9}$.
4. Deduct 29 per cent. from $429·80 and divide the remainder by $10·20.
5. Find the value of $2\frac{3}{4} + 4\frac{5}{11} + \frac{1}{6}$ of $\frac{2}{7}$ of $4\frac{1}{2} + \frac{3}{4} - 5\frac{1}{8}$.
6. What is the simple interest of $943·70 for 11·2 years at 6½ per cent. per annum?
7. Reduce $\cdot\dot{7}$, $\cdot 4\dot{2}$, $\cdot 2\dot{3}5\dot{7}$ and $\cdot\dot{8}7\dot{6}$ to their equivalent vulgar fractions.

8. Bought 729 barrels of flour for $2916, for what must I sell it per barrel in order to gain 28 per cent.?

9. $\frac{7}{8}$ of $\frac{4}{9}$ of $\frac{3}{7}$ of 63 is $\frac{9}{8}$ of how many times 8?

10. Extract the cube root of 86172·1913.

---

11. In 1868 there was exported from Canada $376951 worth of dried and smoked fish, $279404 worth of pickled fish, $19592 worth of fresh fish, and $38936 worth of fish oil; what was the total value of the fish and fish oil exported from Canada in 1868?

12. What is the value of 27 lbs. 4 oz. 6 dwt. 17 grs. multiplied by $629\frac{1}{2}$?

13. Reduce $\frac{14}{48}$, $\frac{121}{847}$, $\frac{28512}{71280}$, and $\frac{19691}{25782}$ to their lowest terms.

14. What is the value of ·714625 of a mile?

15. Divide 90·478 by ·002693.

16. If $\frac{11}{13}$ of a vessel cost $$6294\frac{3}{11}$, what will $\frac{7}{13}$ of the vessel cost?

17. Find the price of 914 lbs. 7 oz. 5 drs. Avoir. at $11·49 per lb.

18. What is the bank discount, and what the true discount on a note for $1106 due 6 months hence, discounting at 7 per cent.?

19. A, B, and C can do a piece of work in 10 days, A working alone can do it in 28 days, and C working alone can do it in 32 days; in what time can B working alone do it?

20. What is the square root of $149\frac{3}{11}$?

---

21. From the upper end of Lake Superior to the mouth of the St. Lawrence is about 2000 miles; what time would a vessel require to make this voyage with an average speed of $6\frac{1}{4}$ miles per hour?

22. What is the difference between £219 8s. $11\frac{1}{2}$d. and $\frac{3}{8}$ of $4\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{1}{3\frac{1}{2}}$ of $24\frac{1}{2}$ times $976·53?

23. Divide 978 acres 2 r. 1 per. 7 yds. by 8 a. 3 r. 27 per. 2 yds.

24. Express 27, 393, 4700, 78904 and 9136718 in Roman Numerals.

25. What is the ratio compounded of 19 : 4, 11 : 5, 12 : $9\frac{1}{4}$, and 33 : 17?

26. Find the G. C. M. of 27051 and 15013.

27. If 4 men or 6 women or 8 boys could do a certain piece of work in 56¼ hours, in what time could 1 man, 2 women, and 5 boys, working together, perform it?

28. 24 men can cut 400 rods of drain 3½ feet deep in 8 days, working 10 hours per day; how many rods of drain 4 feet deep can 32 men cut in 21 days working 9 hours per day?

29. Reduce £789 14s. 8¼d. sterling to dollars and cents, and express $2984·70 in sterling money.

30. What is the amount and compound interest of $800 for 2 years at 4½ per cent. half-yearly?

---

31. Taking the population of Toronto as 50000, that of Hamilton as 22000, that of Kingston as 15000, and the whole population of Canada West as 1600000, what percentage of the entire population of Canada West belongs to each of these cities?

32. Find the l. c. m. of 12, 18, 20, 24, 60, 72, 80, 88, 96, and 100.

33. Express as one number, nine hundred and one trillions forty billions seven thousand and eight, and nine million thirty thousand and seventeen hundredths of trillionths.

34. A, B, and C begin business with a capital of $7900, of which A contributes $2700, B $2300, and C the balance. How should the profits, which amount to $2470, be divided among them?

35. Bought 796 bushels of wheat at $1·20 per bushel and sold the whole for $1000; what was my gain per cent.?

36. Reduce 9146714 inches to acres.

37. Reduce $2967·80 to New England currency, and reduce £29 8s. 11½d. Maryland currency to dollars and cents.

38. Find the value of ·14672 of 17 bushels 1 pk. 1 gal.

39. Extract the cube root of 7149$\frac{3}{11}$.

40. A cistern has two pipes by one of which it can be filled in 40 minutes and by the other in 25 minutes; it has also a discharging pipe by which it can be emptied in 30 minutes. Now if the cistern be empty and all three pipes be opened, in what time will it be filled?

---

41. Divide 71·413 by ·91407 and give the quotient true to three decimal places.

42. Multiply the difference between 43 a. 2 r. 7 per. and 19 a. 3 r. 27 per. 18 yds. by 378.

43. Reduce 278 yds. 8 qr. 1 na. 2 in. into inches.

44. Reduce $\frac{4}{7}$ of $3\frac{1}{2}$ of $6\frac{1}{4}$ of $8\frac{1}{3}$ of $\frac{9\frac{3}{4}}{11\frac{1}{4}}$ of $\frac{8\frac{1}{4}}{17\frac{1}{3}}$ of ·9 of $\frac{3}{8}$ of ·63 to a simple quantity.

45. What is the cost of 23 a. 3 r. 30 per. at $47·80 per acre?

46. What is the worth of $2400 stock in the Great Western Railway when it is selling at a premium of 8½ per cent.?

47. How many paces each 2 feet 5 inches in length will a man make in walking from Hamilton to the Falls of Niagara, a distance of 50 miles?

48. Sold my farm for $7890, which was 16 per cent. more than it cost me; what did I give for it?

49. What is the interest of $894·80 for 7 years, 3 months, 20 days at 6 per cent. per annum?

50. A can do a certain piece of work in 24 days and B can do it in 30 days; now if A works alone at it for 7 days, in what time will A and B working together finish it?

---

51. Express 2 lbs. 3 oz. 4 dwt. as a fraction of 11 lbs. 7 oz. 9 dwt. 4 grs.

52. Divide £493 16s. 4½d. by £8 11s. 7d.

53. Find the l. c. m. of 5, 8, 11, 14, 16, 20, 22, 176, 616, and 42.

54. My agent sells for me 419 barrels of flour at $5·34 per barrel; to what does his commission amount at 11½ per cent.?

55. B has 47 bushels of barley worth 73 cents per bushel and barters it with C for 69 bushels of oats. How much does he allow per bushel for the oats?

56. What must I pay for insuring my house and furniture to the amount of $4250, the premium of insurance being 1½ per cent.?

57. Three persons rent a pasture for $200. A puts in 207 sheep for 4 months, B 109 sheep for 5 months, and C 43 cows for 4½ months; what portion of the rent should each pay, allowing 1 cow to be equivalent to 5 sheep?

58. If a grocer mixes 23 lbs. of tea at 80 cents with 19 lbs. at 75, 30 lbs. at 40, and 42 lbs. at 60 per lb.; what is the worth of 1 lb. of the mixture?

59. Distribute $1100 among A, B, C, and D, so that A may have as much as B, C as much as A and B together, and D as much as A, B, and C together.

60. If 25 men can dig a trench 36 feet long, 12 feet wide, and 6 feet deep in 9 days working 10 hours per day, how many hours a day must 15 men work in order to dig a trench 48 feet long, 8 feet wide, and 5 feet deep, in 12 days?

MENTAL ARITHMETIC.

SUGGESTIONS.

The following exercises will be found to be too difficult, in most instances, for beginners. Before entering them, the pupil is supposed to have been thoroughly drilled in problems of an easier description, as, for example, those given in the body of the book, at the commencement of each of the simple rules, under the head of Mental Exercises. Wherever the teacher finds the exercise too much advanced for his class, he must break down the difficulty by proposing numerous easier questions, of his own construction, involving the same principle.

The recitation should be generally conducted with the *text-books closed.* The teacher should read out the problem slowly and distinctly, and *but once*, and the class should be required to solve it *mentally* and in *perfect silence*, and without giving any sign or signal when they are ready to answer. After a space of time sufficient for the solution of the problem has elapsed, the teacher gives a signal upon which those who have completed the process raise the hand. One of these is then required to give the result. The teacher ascertains how many agree with it, and calls upon some one of them to solve and analyse it for the class. Then another problem is proceeded with in the same manner. Occasionally, especially in review lessons, the class may be allowed to recite with open books.

The pupil should be required to adhere to the form of analysis given, unless the teacher can devise a better for him. In the same school a *uniform phraseology* in the solutions should invariably be adopted.

In order to secure the attention of the entire class, no intimation by word or glance should be given as to the member of the class to be called upon for an answer or solution; so that every one considering himself liable to be selected for that purpose, shall concentrate his mind upon the question.

Finally, the instructor of youth should always remember that Mental Arithmetic, as a branch of school study, is not designed to be merely a means of dazzling and bewildering the adult public by the *rapidity* with which the pupils are trained to solve certain classes of problems, but rather as a mental training of the very highest character. Its object may be said to be threefold, viz.: In the first place, to enable the pupil to solve mentally and with facility the majority of the problems that arise in the business of every-day life, and which otherwise he could work only by tedious processes on his slate; in the second place, to familiarize him with the processes employed in written arithmetic, to render that part of the study of numbers clear to his comprehension, and to make him in a manner independent of mere book-rules; and in the third place and *primarily*, to cultivate his powers of analysis and his ability to concentrate his attention on a given subject,—in a word, to develop and invigorate the most important of his intellectual faculties.

EXERCISE I.

1. How many are 8 and 7? 9 and 12? 8 and 17? 19 and 7? 23 and 11?
2. How many are 12 and 11? 11 and 17? 13 and 14? 17 and 19? 18 and 15?
3. How many are 23 and 71? 38 and 47? 63 and 29? 29 and 81? 47 and 88?
4. How many are 123 and 47? 276 and 93? 489 and 29? 714 and 83?
5. How many are 643 and 293? 216 and 417? 278 and 933? 429 and 276?
6. How many are 1478 and 976? 2913 and 579? 287 and 9163?
7. How many are 4916 and 7189? 9612 and 3407? 9161 and 7863?
8. How many are 19 and 18 and 27?
9. How many are 28 and 143 and 729?
10. How many are 493 and 7816?
11. How many are 9167 and 2347?
12. How many are 7+19+23+47+98+127+246?

EXERCISE II.

1. From 17 take 8 and how many remain? From 81 take 43 and how many remain?
2. From 123 take 48 and how many remain? From 217 take 109 and how many remain?
3. How many are 43—27? 93—42? 67—43? 128—89?
4. How many are 768—400? 293—150? 671—428? 678—434?
5. How many are 47—29? 789—43? 675—71? 891—476?
6. How many are 893—473? 981—671? 493—339? 781—407?
7. How many are 471—89? 96—23? 471—426? 711—189?
8. How many are 8146—23? 7167—93? 9146—217?
9. How many are 8371—986? 6242—555? 9167—8147?
10. How many are 9187—8674? 9321—296? 817—496?
11. How many are 784—27—98—423—11?
12. How many are 9867—2143—478—916—276—43?

EXERCISE III.

1. How many are 9+8+17+43−11+14−72+18+9−17+2?
2. How many are 11−3+8−2+63−47+129−117?
3. How many are 27+43−19−21+87−83+14+29+146−109?
4. How many are 809+476−347−217+401?
5. How many are 9167−434−297−27−64−9+8+91+76+129−718?
6. How many are 1679−816+278−3+78−93+217+411?

EXERCISE IV.

Commit to memory the following :—

EXTENDED MULTIPLICATION TABLE.

13 times	14 times	15 times	16 times	17 times	18 times	19 times
2 are 26	2 — 28	2 — 30	2 — 32	2 — 34	2 — 36	2 — 38
3 — 39	3 — 42	3 — 45	3 — 48	3 — 51	3 — 54	3 — 57
4 — 52	4 — 56	4 — 60	4 — 64	4 — 68	4 — 72	4 — 76
5 — 65	5 — 70	5 — 75	5 — 80	5 — 85	5 — 90	5 — 95
6 — 78	6 — 84	6 — 90	6 — 96	6 — 102	6 — 108	6 — 114
7 — 91	7 — 98	7 — 105	7 — 112	7 — 119	7 — 126	7 — 133
8 — 104	8 — 112	8 — 120	8 — 128	8 — 136	8 — 144	8 — 152
9 — 117	9 — 126	9 — 135	9 — 144	9 — 153	9 — 162	9 — 171

1. How many are 17 times 8? 9 times 14? 11 times 19? 14 times 17?
2. How many are 23 times 9? 7 times 29? 8 times 416? 9 times 217?
3. How many are 16 times 43? 22 times 37? 44 times 29? 19 times 19?
4. How many are 63 times 72? 96 times 43? 34 times 97? 162 times 419?
5. How many are 47 times 46? 35 times 29? 483 times 13? 21 times 498?
6. How many are 717 times 25? 101 times 162? 227 times 463?
7. What is the product of 16 × 91? 17 × 83? 4267 × 8? 7134 × 2?
8. What is the product of 9137 × 8? 21674 × 11? 764 × 25?
9. What is the product of 217 × 17? 4670 × 9? 2703 × 3?
10. What is the product of 9187 × 6? 8885 × 77? 8907 × 64?

EXERCISE V.

1. What is $\frac{1}{5}$ of 9? $\frac{1}{4}$ of 19? $\frac{1}{11}$ of 43? $\frac{1}{13}$ of 67? $\frac{1}{8}$ of 472? $\frac{1}{9}$ of 863?
2. What is $\frac{1}{7}$ of 89? $\frac{1}{9}$ of 8167? $\frac{1}{4}$ of 8149? $\frac{1}{12}$ of 8107? $\frac{1}{8}$ of 2233?
3. What is $\frac{1}{8}$ of 896? $\frac{1}{7}$ of 234? $\frac{1}{9}$ of 89167? $\frac{1}{11}$ of 3871? $\frac{1}{13}$ of 806?
4. What is $\frac{1}{8}$ of 9671? $\frac{1}{8}$ of 587? $\frac{1}{10}$ of 89678? $\frac{1}{17}$ of 83? $\frac{1}{18}$ of 426?
5. What is the quotient of 98 ÷ 7? 463 ÷ 9? 7896 ÷ 8? 9163 ÷ 5?
6. What is the quotient of 278 ÷ 4? 8167 ÷ 11? 671 ÷ 12? 9187 ÷ 13?
7. What is the quotient of 2678 ÷ 8? 6149 ÷ 18? 714 ÷ 20? 914 ÷ 24?
8. What is the quotient of 7786 ÷ 45? 3769 ÷ 27? 8767 ÷ 88?
9. What is the quotient of 7198 ÷ 63? 429 ÷ 19? 6789 ÷ 14?
10. What is the quotient of 123459 ÷ 96? 67143 ÷ 90? 217654 ÷ 77?

EXERCISE VI.

1. 8 times 9 and $\frac{7}{9}$ of 9 are how many times 10?

ANALYSIS.

8 times 9 are 72, and $\frac{7}{9}$ of 9 is 7 times $\frac{1}{9}$ of 9; $\frac{1}{9}$ of 9 is 1, therefore $\frac{7}{9}$ of 9 is 7 times 1, which is 7.

Therefore 8 times 9 and $\frac{7}{9}$ of 9 are 72 and 7 which make 79.

Then 10 is contained $7\frac{9}{10}$ times in 79.

Therefore 8 times 9 and $\frac{7}{9}$ of 9 are $7\frac{9}{10}$ times 10.

2. 4 times 5 and $\frac{3}{5}$ of 5 are how many times 6? 7? 8?
3. 7 times 11 and $\frac{9}{11}$ of 11 are how many times 7? 8? 10?
4. 6 times 8 and $\frac{7}{8}$ of 8 are how many times 3? 5? 9?
5. 11 times 3 and $\frac{4}{9}$ of 9 are how many times 10? 9? 4?
6. 10 times 5 and $\frac{3}{5}$ of 25 are how many times 7? 8? 10?
7. 8 times 7 and $\frac{4}{7}$ of 28 are how many times 9? 11? 12?
8. 12 times 9 and $\frac{7}{9}$ of 63 are how many times 10? 11? 13?
9. 8 times 6 and $\frac{5}{6}$ of 30 are how many times $\frac{1}{4}$ of 16? $\frac{3}{5}$ of 15?
10. 7 times 4 and $\frac{3}{4}$ of 44 are how many times $\frac{3}{7}$ of 21? $\frac{5}{11}$ of 55?

EXERCISE VII.

1. What is $\frac{3}{7}$ of that number of which 12 is $\frac{6}{11}$?

ANALYSIS.

If 12 is $\frac{6}{11}$ of a certain number, $\frac{1}{11}$ will be the $\frac{1}{6}$ of 12, which is 2.

If 2 is $\frac{1}{11}$ of a certain number, 11 times 2, which is 22, will be that number.

Then $\frac{3}{7}$ of 22 is equal to $\frac{1}{7}$ of $22 \times 3 = 3\frac{1}{7} \times 3 = 9\frac{3}{7}$.

Therefore $9\frac{3}{7}$ is $\frac{3}{7}$ of that number of which 12 is $\frac{6}{11}$.

2. What is $\frac{4}{11}$ of that number of which 21 is $\frac{3}{5}$?
3. What is $\frac{2}{9}$ of that number of which 81 is $\frac{3}{8}$?
4. What is $\frac{3}{8}$ of that number of which 36 is $\frac{4}{11}$?
5. What is $\frac{4}{7}$ of that number of which 18 is $\frac{6}{7}$?
6. What is $\frac{4}{5}$ of that number of which 51 is $\frac{17}{18}$?
7. What is $\frac{7}{8}$ of that number of which 77 is $\frac{11}{14}$?
8. What is $4\frac{3}{8}$ times that number of which 80 is $\frac{8}{11}$?

EXERCISE VIII.

1. 25 is $\frac{5}{7}$ of how many times 9?

ANALYSIS.

If 25 is $\frac{5}{7}$ of a certain number, $\frac{1}{7}$ will be $\frac{1}{5}$ of 25, which is 5.

If $\frac{1}{7}$ of a number is 5, the number must be 5 × 7, which is 35.

Then $35 \div 9 = 3\frac{8}{9}$.

Therefore 25 is $\frac{5}{7}$ of $3\frac{8}{9}$ times 9.

2. 84 is $\frac{7}{8}$ of how many times 10? 7? 9? 11?
3. 63 is $\frac{9}{11}$ of how many times 7? 8? 5? 12?
4. 21 is $\frac{7}{8}$ of how many times 11? 6? 5?
5. 96 is $\frac{8}{9}$ of how many times 5? 7? 13?
6. 121 is $\frac{11}{12}$ of how many times 12? 10?
7. 105 is $\frac{5}{8}$ of how many times 6? 11?

EXERCISE IX.

1. $\frac{9}{11}$ of 44 is $\frac{7}{10}$ of how many ninths of 54?

ANALYSIS.

$\frac{9}{11}$ of 44 is 9 times $\frac{1}{11}$ of 44; $\frac{1}{11}$ of 44 is 4, therefore $\frac{9}{11}$ of 44 is 9 times 4, which is 36. Also $\frac{1}{9}$ of 54 is 6.

Then 36 is $\frac{7}{10}$ of how many 6's.

If 36 is $\frac{7}{10}$ of a certain number, $\frac{1}{10}$ will be $\frac{1}{7}$ of 36, which is $5\frac{1}{7}$; and if $5\frac{1}{7}$ is $\frac{1}{10}$ of a certain number, that number will be 10 times $5\frac{1}{7}$ which is $51\frac{3}{7}$.

Then $51\frac{3}{7}$ divided by 6 equals $8\frac{4}{7}$.

Therefore $\frac{9}{11}$ of 44 is $\frac{7}{10}$ of $8\frac{4}{7}$ times $\frac{1}{9}$ of 54.

2. $\frac{4}{9}$ of 54 is $\frac{4}{5}$ of how many times $\frac{3}{8}$ of 16?
3. $\frac{7}{8}$ of 48 is $\frac{6}{9}$ of how many times $\frac{4}{7}$ of 91?
4. $\frac{6}{11}$ of 77 is $\frac{7}{8}$ of how many times $\frac{9}{11}$ of 88?
5. $\frac{4}{7}$ of 91 is $\frac{9}{11}$ of how many times $\frac{3}{5}$ of 30?
6. $\frac{5}{8}$ of 104 is $\frac{4}{7}$ of how many times $\frac{7}{8}$ of 56?
7. $\frac{7}{9}$ of 63 is $\frac{6}{7}$ of how many times $\frac{9}{10}$ of 150?
8. $\frac{4}{11}$ of 121 is $\frac{11}{12}$ of how many times $\frac{6}{7}$ cf 21?

EXERCISE X.

1. What will be the cost of 17 sheep at \$4·20 each?
2. What will be the cost of 11 horses at \$79·80 each?
3. The sum of two numbers is 493; the greater is 284, what is the smaller?
4. The multiplier is 40, the product 9840, what is the multiplicand?
5. What is the difference between \$278·80 and \$127·63?
6. What is the ninth part of \$2987·80?
7. What is the product of 783 × 72?
8. How many are $\frac{4}{9}$ of 639?
9. 8 times 5 and $\frac{3}{5}$ of 20 are how many times 7? 8? 9?
10. 7 times 11 and $\frac{6}{11}$ of 33 are how many times 12? 10? 8?
11. What is $\frac{1}{4}$ of $\frac{2}{3}$ of that number of which 34 is $\frac{17}{6}$?

12. What is $\frac{3}{8}$ of $\frac{3}{7}$ of that number of which 27 is $\frac{9}{11}$?
13. 42 is $\frac{7}{8}$ of how many times 5? 7? 11?
14. $\frac{6}{11}$ of 55 is $\frac{7}{8}$ of how many times $\frac{5}{6}$ of 18?
15. $\frac{1}{5}$ of 4 times $6\frac{1}{4}$ is $\frac{3}{7}$ of how many times $\frac{3}{5}$ of $\frac{2}{3}$ of 2 times 15?
16. $\frac{7}{11}$ of 6 times $9\frac{1}{6}$ is $\frac{5}{8}$ of how many times $\frac{2}{7}$ of $\frac{1}{3}$ of 5 times $8\frac{2}{5}$?

EXERCISE XI.

1. If $\frac{2}{3}$ of a barrel of apples cost \$1·80, what is that per barrel?

ANALYSIS.

If $\frac{2}{3}$ cost \$1·80, $\frac{1}{3}$ will cost $\frac{1}{2}$ of \$1·80, which is \$0·90.

If $\frac{1}{3}$ cost \$0·90, the whole barrel will cost 3 times \$0·90, which is \$2·70.

Therefore if $\frac{2}{3}$ of a barrel of apples cost \$1·80, the whole barrel will cost \$2·70.

2. If $\frac{4}{11}$ of a lb. of tea cost 44 cents, what will 1 lb. cost?
3. If $\frac{3}{4}$ of a day's work cost 87 cents, to what will 4 days' work amount?
4. If $\frac{1}{3}$ of 6 lbs. of coffee cost $\frac{5}{9}$ of \$2, what will $\frac{8}{11}$ of 5 lbs. cost?
5. How much will $\frac{3}{4}$ of a barrel of flour cost, if $\frac{2}{3}$ cost \$1·60?
6. How much will a basket of peaches cost, if $\frac{1}{3}$ cost \$2·70?
7. How much will 4 stone of meal come to if $\frac{2}{3}$ of a stone cost 23 cents?
8. How much will 6 cords of wood amount to if $\frac{2}{11}$ of 2 cords cost \$2·20?

EXERCISE XII.

NOTE.—*The teacher must thoroughly explain how fractions are added, subtracted, reduced, multiplied, and divided.*

1. What is the quotient of $7\frac{1}{5} \div 6\frac{3}{4}$?

SOLUTION.

$7\frac{1}{5} = \frac{36}{5}$ and $6\frac{3}{4} = \frac{27}{4}$. Then $7\frac{1}{5} \div 6\frac{3}{4} = \frac{36}{5} \div \frac{27}{4} = \frac{36}{5} \times \frac{4}{27} = \frac{4}{5} \times \frac{4}{3} = \frac{16}{15} = 1\frac{1}{15}$.

2. What is the value of $\frac{5}{8} + \frac{1}{24} - \frac{1}{3}$?

SOLUTION.

$\frac{5}{8} = \frac{15}{24}$ and $\frac{1}{3} = \frac{8}{24}$. Then $\frac{5}{8} + \frac{1}{24} - \frac{1}{3} = \frac{15}{24} + \frac{1}{24} - \frac{8}{24} = \frac{16}{24} - \frac{8}{24} = \frac{8}{24} = \frac{1}{3}$. *Ans.*

3. How many are $\frac{4}{7}$ of $\frac{8}{11}$? $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{4}{7}$? $\frac{2}{3}$ of $\frac{3}{11}$ of $\frac{77}{100}$ of 25? $\frac{1}{3}$ of $\frac{3}{5}$ of $\frac{19}{27}$ of $\frac{18}{47}$?

4. What is the value of $\frac{3}{5}+\frac{2}{7}$? $\frac{3}{8}+\frac{9}{11}$? $\frac{7}{8}+\frac{5}{16}$? $9\frac{1}{3}+8\frac{1}{4}$? $16\frac{3}{8}+8\frac{5}{12}$? $2\frac{1}{3}+3\frac{4}{7}$?

5. What is the value of $\frac{9}{11}-\frac{4}{11}$? $7\frac{1}{3}-2\frac{1}{4}$? $6\frac{3}{4}-2\frac{7}{16}$? $11\frac{1}{3}-7\frac{3}{8}$? $27\frac{8}{11}-4\frac{7}{13}$?

6. $6\frac{3}{4}$ are how many fourths? $2\frac{3}{7}$ are how many sevenths? $5\frac{7}{11}$ are how many elevenths?

7. $4\frac{3}{7}$ is $\frac{1}{11}$ of what number? $2\frac{3}{8}$ is $\frac{7}{13}$ of what number? $5\frac{3}{7}$ is $\frac{3}{8}$ of what number?

8. 13 is 6 times what number? 11 is 4 times what number? 17 is 12 times what number?

9. 63 is $\frac{4}{11}$ of what number? 29 is $\frac{7}{8}$ of what number? 16 is $\frac{4}{7}$ of what number?

10. What is the product of $\frac{1}{3}\times\frac{3}{7}\times\frac{4}{5}\times\frac{14}{17}\times\frac{34}{15}$? $6\frac{2}{3}\times\frac{17}{22}$? $9\frac{3}{4}\times\frac{1}{13}$?

11. What is the quotient of $8\frac{3}{4}\div\frac{1}{4}$ of $\frac{3}{4}$ of $\frac{5}{8}$ of 21? $7\div\frac{3}{8}$ of $\frac{8}{11}$?

12 What is the value of $\frac{4}{7}\times\frac{4}{11}\div\dfrac{3}{57\frac{1}{3}}$? $6\frac{3}{7}\times\frac{1}{15}\div8\frac{1}{4}$?

EXERCISE XIII.

1. If 11 sheep cost $37, what should 9 sheep cost?

ANALYSIS.

Since 11 sheep cost \$37, 1 sheep should cost $\frac{1}{11}$ of \$37, which is \$$3\frac{4}{11}$; and if 1 sheep cost \$$3\frac{4}{11}$, 9 sheep should cost 9 times \$$3\frac{4}{11}$, which is \$$30\frac{3}{11}$.

Therefore if 11 sheep cost \$37, 9 sheep should cost \$$30\frac{3}{11}$.

2. If 8 cords of wood cost 27 dollars, what will 17 cords cost?
3. If 3 barrels of flour cost 22 dollars, what will 11 barrels cost?
4. If 7 days' work amount to 17 dollars, to what will 3 days' work amount?
5. If 9 acres of land cost 57 dollars, what will 13 acres cost?
6. If 11 men do a piece of work in 40 days, in how many days can 7 men do it?
7. If $8\frac{3}{4}$ tons of hay be bought for \$105, what would be the cost of $9\frac{3}{11}$ tons?

8. At \$7 for 11 bushels of barley, what would be the cost of 21 bushels?
9. At 6 lbs. of tea for \$5, how many lbs. can be had for \$23?
10. If \$8 pay for 7 days' work, for how many days will \$29 pay?

EXERCISE XIV.

1. If 3 horses consume $8\frac{3}{7}$ bushels of oats in 2 weeks, how many bushels would 5 horses consume in 3 weeks?

ANALYSIS.

If 3 horses consume $8\frac{3}{7}$ bushels, one horse will consume $\frac{1}{3}$ of $8\frac{3}{7}$, which is $2\frac{17}{21}$ bushels. If 1 horse consume $2\frac{17}{21}$ bushels in 2 weeks, in one week he will consume $\frac{1}{2}$ of $2\frac{17}{21}$, which is $1\frac{17}{42}$. If a horse consume $1\frac{17}{42}$ bushels in 1 week, in 3 weeks he will consume 3 times $1\frac{17}{42}$, which is $4\frac{3}{14}$ bushels; and if one horse consume $4\frac{3}{14}$ bushels, 5 horses will consume 5 times $4\frac{3}{14}$, which is $21\frac{1}{14}$ bushels. Therefore, &c.

2. If a certain quantity of provisions would last 16 men $4\frac{3}{4}$ days, how long would 4 times the quantity last $\frac{3}{4}$ the number of men?
3. If 5 men can accomplish a piece of work in $7\frac{3}{4}$ days, in what time can 3 men finish $\frac{2}{3}$ of the work?
4. If 7 men in 4 days working 10 hours per day can build a certain length of wall, how many hours per day must 11 men work in order to build the same length in 3 days?
5. If \$60 pay 7 men for 9 days' work, how many dollars will pay 11 men for 11 days' work?
6. If 24 men can mow 66 acres of grass in 2 days, how many acres can 14 men mow in 7 days?

EXERCISE XV.

1. What number is that of which $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ added together make 55?

ANALYSIS.

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{30}{60} + \frac{20}{60} + \frac{15}{60} + \frac{12}{60} = \frac{77}{60}.$$

If $\frac{77}{60}$ of a certain number is 55, $\frac{1}{60}$ of the number will be $\frac{1}{77}$ of 55, which is $\frac{55}{77}$ or $\frac{5}{7}$.

If $\frac{1}{60}$ be $\frac{5}{7}$, the number itself will be 60 times $\frac{5}{7}$, which is $42\frac{6}{7}$. Therefore if 55 is the sum of $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{4}$, and $\frac{1}{2}$, the number itself is $42\frac{6}{7}$.

2. Having counted my books, I found that $\frac{1}{3}$, $\frac{2}{5}$, and $\frac{1}{4}$ added together amounted to 590 ; how many had I ?

3. After paying away $\frac{3}{7}$ of my money and then $\frac{3}{5}$ of the remainder, I had $80 remaining : how much had I at first ?

4. If to $\frac{3}{4}$ of B's age you add 14 years, the sum will be $1\frac{1}{24}$ times his age ; how old is he ?

5. If from $\frac{7}{11}$ of C's age you subtract $5\frac{1}{2}$ years, the remainder will be $\frac{31}{66}$ of his age : how old is he ?

6. If to $\frac{3}{7}$ of the cost of my house you add $200, the sum will be $\frac{1}{2}$ the cost of my house ; what was the cost of the house ?

7. If from my age you subtract $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{6}$ of my age, the remainder will be $1\frac{1}{3}$ years ; how old am I ?

8. A boy being asked his age, replied that it was 2 years more than $\frac{1}{5}$ of his mother's age, and that 12 years before that time his mother was 2 years more than half as old as her father, who was then 72 years of age ; how old was the boy ?

EXERCISE XVI.

1. A can do a piece of work in $4\frac{1}{2}$ days which B can do in $5\frac{3}{7}$ days ; in what time could they do it working together ?

ANALYSIS.

If A can do the whole work in $4\frac{1}{2}$ days, in 1 day he would do $\frac{2}{9}$ of it ; and if B can do the whole work in $5\frac{3}{7}$ days, in 1 day he would do $\frac{7}{38}$ of it.

Then since A does $\frac{2}{9}$ and B $\frac{7}{38}$ of the work in 1 day, working together, they would do $\frac{2}{9} + \frac{7}{38}$, which is $\frac{139}{342}$.

Then, if they do $\frac{139}{342}$ in 1 day, they will require as many times 1 day as $\frac{139}{342}$ is contained times in 1, which is $\frac{342}{139}$, and this is equal to $2\frac{64}{139}$ days. Therefore, &c.

2. A can do a piece of work in 7 days which B can do in 6 days, in what time will they do it working together ?

3. A can dig a certain garden in 4 days which B can do in 6 and C in 8 days ; in what time will they finish it working together ?

4. A can do a piece of work in 30 days which B can do in 25, C in 20, and D in 15 days ; in what time will they finish it working together ?

5. A and B can cradle a field of wheat in 10 days, which A alone could do in 17 days ; in what time could B alone do it ?

6. A, B, and C can finish a certain amount of work in 20 days which A could do alone in 40 days and C in 60 days; in what time could B working alone finish it?

Exercise XVII.

1. What is the value of 7896 bushels of oats at 37½ cents per bushel?

ANALYSIS.

37½ cents is ⅜ of $1; 7896 bushels at $1 would amount to $7896. Then, since 37½ cents is ⅜ of $1, 7896 bushels at 37½ will amount to ⅜ of $7896; ⅜ of $7896 is 3 times ⅛ of $7896, ⅛ of $7896 is $987, and 3 times $987 is $2961. Therefore, &c.

2. What is the value of 798 bushels of wheat at $1·50 per bushel?
3. What is the value of 916 lbs. of tea at 75 cents per lb.?
4. What is the value of 2718 yds. of cotton at 20 cents per yd.?
5. What is the value of 3612 doz. eggs at 16⅔ cents per doz.?
6. What is the value of 5780 yds. of linen at 87½ cents per yd.?
7. What is the value of 7567 bush. of oats at 50 cents per bush.?
8. What is the value of 719 days' work at 33⅓ cents per day?
9. What is the value of 5796 yds. of drugget at 62½ cents per yd.?
10. What is the value of 478 ft. of chesnut lumber at 5 cents per foot?
11. What is the value of 7864 lbs. of butter at 12½ cents per lb.?
12. What is the value of 1160 bushels of turnips at 40 cents per bushel?

Exercise XVIII.

1. How much is 8⅓ per cent. of $949?

ANALYSIS.

8⅓ per cent. is $\frac{8\frac{1}{3}}{100}$, which is $\frac{1}{12}$, and $\frac{1}{12}$ of $949 is 79\frac{1}{12}$.
Therefore, &c.

2. How much is 20 per cent. of $555·50?
3. How much is 66⅔ per cent. of $540·83?
4. How much is 10 per cent. of $89?
5. How much is 12½ per cent. of $978?
6. How much is 50 per cent. of $429·60?
7. How much is 6¼ per cent. of $727·20?
8. How much is 25 per cent. of $606·40?

EXERCISE XIX.

1. What is the premium of insurance on $764 at 8 per cent.?

ANALYSIS.

8 per cent. is 8 times one per cent.
1 per cent. of $764 is $7·64.
Then 8 per cent. of $764 is $7·64 × 8, which is $61·12.

2. What is the commission on $480 at 11 per cent.?
3. What is the brokerage on $9806 at 12 per cent.?
4. What is the brokerage on $5980 at 2 per cent.?
5. What is the commission on $578 at 18 per cent.?
6. What is the commission on $789 at 30 per cent.?
7. What is the commission on $5986 at 7 per cent.?
8. What is the premium of insurance on $7890 at 2 per cent.?
9. What is the premium of insurance on $860 at 4 per cent.?
10. What is the premium of insurance on $6790 at 3 per cent.?

EXERCISE XX.

1. At 4 per cent. for 7½ years what fraction is the interest of the principal?

ANALYSIS.

If the interest for one year be $\frac{4}{100}$, for 7½ years it will be 7½ times $\frac{4}{100}$, which is $\frac{30}{100}$ or $\frac{3}{10}$.

Therefore at 4 per cent. for 7½ years the interest equals $\frac{3}{10}$ of the principal.

2. At 5 per cent. for 5 years what fraction of the principal is the interest?

3 At 8 per cent. for 3¾ years what fraction is the interest of the principal?

4 At 8 per cent. for 5 years what fraction of the principal is the interest?

5. At 7 per cent. for 10 years what fraction is the interest of the principal?

6. At 9½ per cent. for 4 years what fraction is the interest of the principal?

7 At 6 per cent. for 4 years 2 months what fraction is the interest of the principal?

8 At 8⅓ per cent. for 6 years what fraction is the interest of the principal?

EXERCISE XXI.

1. What is the interest of $748·60 for 8 years 4 months at 6 per cent.?

ANALYSIS.

8 yrs. 4 m. = $8\frac{1}{3}$ years and $8\frac{1}{3} \times \frac{6}{100} = \frac{50}{100} = \frac{1}{2}$, and hence the interest is equal to $\frac{1}{2}$ of the principal: $\frac{1}{2}$ of $748·60 is $374·30.

Therefore $374·30 is the interest of $748·60 for 8 yrs. 4 months at 6 per cent.

2. What is the interest of $467·84 for 12 years 6 months at 8 per cent.?
3. What is the interest of $916·70 for 5 years at 5 per cent.?
4. What is the interest of $543·20 for 3 years at 10 per cent.?
5. What is the interest of $943 for 4 years at $12\frac{1}{2}$ per cent.?
6. What is the interest of $789 for 3 yrs. 4 m. at 3 per cent.?
7. What is the interest of $47·23 for 7 years at $2\frac{6}{7}$ per cent.?
8. What is the interest of $47·89 for 8 years 4 months at 3 per cent.?
9. What is the interest of $896·80 for $9\frac{1}{11}$ years at 11 per cent.?
10. What is the interest of $1027·40 for 4 years at $2\frac{1}{2}$ per cent.?

EXERCISE XXII.

1. What is the interest of $560 for 5 years at 7 per cent.?

ANALYSIS.

7 per cent. is $\frac{7}{100}$ per unit and $\frac{7}{100} \times 5 = \frac{35}{100} = \frac{7}{20}$; hence the interest is $\frac{7}{20}$ of the principal, that is 7 times $\frac{1}{20}$; $\frac{1}{20}$ of $560 is $28. Therefore the interest is $28 × 7, which is $196.

2. What is the interest of $840 for 8 years at 10 per cent.?
3. What is the interest of $1100 for 7 years at 20 per cent.?
4. What is the interest of $700 for $9\frac{1}{2}$ years at 4 per cent.?
5. What is the interest of $640 for 8 years at 8 per cent.?
6. What is the interest of $500 for 7 years at 7 per cent.?
7. What is the interest of $1000 for 4 years at $6\frac{1}{4}$ per cent.?
8. What is the interest of $890 for 6 years at 5 per cent.?
9. What is the interest of $720 for 9 years at 4 per cent.?
10. What is the interest of $880 for 2 years at $5\frac{1}{2}$ per cent.?

EXERCISE XXIII.

1. What is the interest of $108 for 1 year 2 m. at 6 per cent.?

ANALYSIS.*

The interest of $1 for 14 months at 6 per cent. is 7 cents.

Therefore the interest on $108 will be 108 times 7 cents or 7 times 108 cents or 7 times $1·08 cents, which is $7·56.

2. What is the interest of $700 for 17 months at 6 per cent.?
3. What is the interest of $890 for 2 years 4 months at 6 per cent.?
4. What is the interest of $763 for 3 years 4 months at 6 per cent.?
5. What is the interest of $420 for 5 years 2 months at 6 per cent.?
6. What is the interest of $810 for 5 months at 6 per cent.?
7. What is the interest of $703 for 7 months at 3 per cent.?
8. What is the interest of $809 for 11 months at 12 per cent.?
9. What is the interest of $670 for 8 months at 18 per cent.?
10. What is the interest of $893 for 4 months at 8 per cent.?

EXERCISE XXIV.

1. What principal will in 6 years at $3\frac{1}{3}$ per cent. amount to $720?

ANALYSIS.

At $3\frac{1}{3}$ per cent. for six years the interest is $\frac{1}{5}$ of the principal, and the amount, which is equal to the principal added to the interest, is equal to $\frac{5}{5} + \frac{1}{5} = \frac{6}{5}$ of the principal.

If $\frac{6}{5}$ of the principal is $720, $\frac{1}{5}$ of the principal is $\frac{1}{6}$ of $720, which is $120; and if $120 is $\frac{1}{5}$, the whole principal is $120 × 5, which is $600.

Therefore, &c.

2. What principal will in 8 years 4 m. at 6 per cent. amount to $206?
3. What principal will in $9\frac{1}{3}$ years at 9 per cent. amount to $76
4. What principal will in 4 years at 5 per cent. amount to $468?
5. What principal will in $3\frac{4}{7}$ years at 7 per cent. amount to $555·55?

---

* See Rule, page 151.

6. What principal will in 6 years 3 m. at 8 per cent. amount to $573·40?
7. What principal will in 8 years at 5 per cent. amount to $735?
8. What principal will in 7 years at 7½ per cent. amount to $843·20?
9. What principal will in 12 years 6 m. at 8 per cent. amount to $643·28?
10. What principal will in 1 year at 33⅓ per cent. amount to $963·24?
11. What principal will in 7 years at 10 per cent. amount to $561?
12. What principal will in 5 years at 5 per cent. amount to $678·20?

EXERCISE XXV.

1. Bought a cow for $25 and sold it for $29: what was my gain per cent.?

SOLUTION.

Since I bought for $25 and sold for $29, my gain was $29 — $25, which is $4.

If I gain $4 on $25, on $1 I shall gain $\frac{1}{25}$ of $4, which is $\frac{4}{25}$ of a dollar; and if I gain $\$\frac{4}{25}$ on $1, on $100 I shall gain 100 times $\$\frac{4}{25}$, which is $16. Therefore my gain is $16 on $100, or 16 per cent.

2. Bought barrels at 12 cents each and sold at 15; what was my gain per cent.?
3. Bought butter at 15 cents per lb. and sold it at 21 cents; what was my gain per cent.?
4. Bought a cart for $45 and sold it for $50; what was my gain per cent.?
5. Bought wood at $3·50 and sold it at $4·25 per cord; what was my gain per cent.?
6. Bought oats at 38 cents and sold them at 44 cents per bushel, what was my gain per cent.?
7. Bought hay at $18 and sold it at $16 per ton; what was my loss per cent.?
8. Bought a horse for $160 and sold it for $145; what was my loss per cent.?
9. Bought wheat at $1·20 and sold it at $1·50; what was my gain per cent.?
10. Bought lumber at $9·50 and sold it at $11·25 per 1000 ft.; what was my gain per cent.?

ANSWERS TO THE EXERCISES.

EXERCISE 4.

1. 9 ; 18 ; 33 ; 44 ; 99 ; 478 ; 330 ; 47 ; 74.
2. 777 ; 296 ; 843 ; 910 ; 1001 ; 1400.
3. 102 ; 511 ; 1539 ; 3030 ; 2858 ; 301.
4. 333 ; 10000 ; 90000 ; 6000 ; 5977; 27027 ; 40444.
5. 1899 ; 2222 ; 4505 ; 9604 ; 8888.
6. CCII ; XLVII ; XCI ; LXXX ; XX ; LXXVII ; CI ; X ; CXI ; DCVI.
7. CDXXXVII ; CMVIII ; DCCCXCIX ; DCCLXIII ; CDXCVII ; DCCCXXIX ; DCCCXXVII ; CMXCIX ; DCCCLXXXVIII.
8. MMCCXXXIII ; MMMCCXXXII ; MMMCCCXXXIII ; $M\overline{V}CCCXXI$; MCCXXXIV ; $\overline{V}DCLXXVIII$; $\overline{V}MMMDCCLXV$.
9. $M\overline{X}CMXCIX$; $\overline{XX}VDCLXXI$; $\overline{DCCCXCM}CCCXLVII$; $\overline{CMX}MMCCCXLII$; $\overline{XV}MDCCXIII$.
10. $\overline{CXC}MCMXIX$; $\overline{XX}M\overline{X}CXXXIV$; $\overline{XX}MMMCDLXXVI$; $\overline{CMX}MMCCCXLV$; $\overline{MDCLXXV}MMMCMXLII$; $\overline{MMMCDLV}MDCCXIII$.

EXERCISE 5.

1. 879.
2. 8785.
3. 9536.
4. 6424.
5. 38904.
6. 997688.
7. 796.
8. 25734.
9. 5647.
10. 95878.
11. 89745.
12. 3499.

EXERCISE 6.

1. 189930.
2. 19168.
3. 2062862.
4. 203883.
5. 176258.
6. 63665.
7. 181220.
8. 113092.
9. 1171791.
10. 3726390.
11. 670614.
12. 1108958.
13. 1563804.
14. 28140244.
15. 32287760.
16. 5586789.
17. 98536.
18. 172686.
19. 11347.
20. 26132.
21. 46429.
22. 345482.
23. 746456.
24. 1069843784.
25. 864.
26. 320 bushels.
27. $29431920.
28. 20213174 bushels.
29. 4035 bushels.
30. 8638 prs. of shoes.
31. In the year 1877.

EXERCISE 7.

1. 51144002.
2. 3410530.
3. 600002.
4. 1050.
5. 9333333.
6. 109113.
7. 702512.
8. 611803.
9. 204104053.
10. 400340.

EXERCISE 8.

1. 664072.
2. 802909.
3. 10817.
4. 172723.
5. 7253073.
6. 1719034.
7. 730139.
8. 8084031.
9. 70850871.
10. 4613839.
11. 950017022.
12. 581259463.
13. 61374.
14. 701353.
15. 406082.
16. 64021479.
17. 3389380.
18. 6342920.
19. 384.
20. 18294.
21. 1260000.
22. 111.
23. $1072.
24. $662.
25. $3820.
26. 922.
27. 26993[illegible]8.
28. 365000.
31. 68024.
32. 71152.
33. 2644.
34. 2585.
37. 744988
38. 56867.
39. 796098
41. 717258
42. 28518.
43. 632967.

EXERCISE 9.

1. 142982742.
2. 275456841.
3. 364538876.
4. 335719585.
5. 5512592526.
6. 6154158486.
7. 98863888.
8. 8884480482.
9. 16714913450.
10. 86176405301.
11. 51497984052.
12. 629309344.
13. 2375628 ; 1583752 ; 3167504 ; 9502512.
14. 5730338029 ; 6548958776 ; 7367578623 ; 9004818317.
15. 18537 ; 30895 ; 43253 ; 55611 ; 74148.
16. 69871590, 48910113; 55897272; 41922954.
17. 8576628.
18. 17221284.
19. 612784.
20. 4400078056.
21. 4998 ; 7854; 4284 ; 3570; 8568.

EXERCISE 10.

1 81553016.
2. 77008136.
3. 102[illegible]7900.
4. 2[illegible]874[illegible]4.
5 5807538.
6. 9909637.
7. 103235610.
8. 9020046.
9. 8058876.
10. 57054464.
11. 99249624.
12. 101037948.
13. 43529088.
14. 439455324.
15. $7208.
16. $3024.
17. $78900.
18. 969024.
19. 37873.

EXERCISE 11.

1. 165404178.
2. 72532864.
3. 572161070.
4. 7099800912.
5. 52588916400.
6. 57857715882.
7. 772820915328.
8. 2607973180020.
9. 64311091624800.
10. 12107036926863.
11. 82660927680268.
12. 89432071615610683
13. 656650012.
14. 749188440.
15. 656178570630.
16. 2964622653999.
17. 131141615220.
18. 11017864680.
19. 1893398780.
20. 4321 bushels.
21. $23002.
22. $95367.
23. 115200 perches.
24. 501296.
25. 2977593.
26. 1081.

EXERCISE 12.

1. $3573493\frac{1}{3}$.
2. $2971522\frac{1}{3}$.
3. $228524\frac{1}{2}$.
4. $26009439\frac{2}{3}$.
5. $164608405\frac{2}{9}$.
6. $155924451\frac{6}{7}$.
7. 30602382.
8. 10163522.
9. $22233344\frac{4}{10}$.
10. $283755923\frac{8}{11}$.
11. $76213056\frac{8}{13}$.
12. $12345679012\frac{3}{9}$.
13. $4573145\frac{1}{3}$.
14. $238210\frac{3}{5}$.
15. $308652\frac{3}{4}$.
16. 1400000.
17. $1350067\frac{4}{9}$.
18. 1285947.
19. $12576258\frac{3}{4}$.
20. $9999999\frac{9}{10}$.
21. $7407407\frac{6}{15}$.
22. 13717421.
23. $83479685\frac{8}{11}$.
24. $28571428\frac{4}{7}$.
25. $16727475\frac{11}{18}$.
26. $126.
27. $67\frac{9}{11}$ bushels.
28. 161.

EXERCISE 13.

1. $44685\frac{7}{15}$.
2. $3737\frac{2}{37}$.
3. $113416\frac{17}{61}$.
4. $112164\frac{96}{145}$.
5. $39807\frac{36}{42}$.
6. $205761\frac{34}{108}$.
7. $5100407\frac{77}{131}$.
8. $81828384\frac{55}{100}$.
9. $10434202\frac{53}{64}$.
10. $1462940351\frac{44}{56}$.
11. $8336800\frac{6}{110}$.
12. $1111174\frac{57}{63}$.
13. $580662\frac{39}{56}$.
14. $1191\frac{36}{60}$ bushels.
15. $16404\frac{30}{58}$ bushels.
16. $6077\frac{31}{45}$ bushels.
17. $$36\frac{18}{46}$.
18. 60 lbs.
19. $128909\frac{30}{68}$.
20. $$1588\frac{38}{45}$.

EXERCISE 14.

1. $39280\frac{166}{247}$.
2. $7313\frac{105}{1115}$.
3. $711\frac{1340}{2108}$.
4. $1007\frac{4673}{8101}$.
5. $310536\frac{19}{23}$.
6. $363006\frac{10}{17}$.
7. $315\frac{21163}{45181}$.
8. $3356\frac{880}{5344}$.
9. $1134\frac{67488}{91007}$.

10. $1002\frac{4677}{10123}$.
11. $828\frac{4016}{11067}$.
12. $2867\frac{96}{3143}$.
13. $576\frac{3624}{14161}$.
14. 91874163.
15. 8050.
16. 9258.
17. $328\frac{715}{917}$.
18. $3860\frac{28}{61}$.
19. $205\frac{33}{1037}$.
20. $847\frac{3167}{4477}$.
21. $42\frac{28}{317}$ days.
22. $836\frac{13}{43}$ acres.
23. $6722\frac{1735}{4017}$.
24. $198328\frac{33666}{40600}$.
25. $8865\frac{20351}{76410}$.
26. $$23\frac{10}{13}$.

EXERCISE 15.

1. $$2879{\cdot}27\frac{1}{3}$.
2. $$2686{\cdot}53\frac{1}{3}$.
3. $$668{\cdot}17\frac{1}{13}$.
4. $$71{\cdot}52\frac{1}{3}$.
5. $$2623{\cdot}94\frac{7}{13}$.
6. $3110·25.
7. $$446{\cdot}38\frac{1}{3}$.
8. $$2269{\cdot}75\frac{5}{13}$.
9. $$229{\cdot}69\frac{1}{3}$.
10. $$2819{\cdot}99\frac{7}{13}$.

EXERCISE 16.

1. £179 17s.; £229 5s. 6d.; £17 8s. 6d.; £104 0s. 9d.
2. £42 15s. 6¾d.; £47 10s. 5¾d.; £418 11s. 1¼d.; £194 8s. 10¼d.
3. £111 2s. 2¾d.; £27 16s. 3¾d.; £17 10s. 4¼d.; £47 19s. 1¼d.
4. £178 11s. 1¼d.; £5 5s. 10¼d.; £4 0s. 9¾d.; £1983 14s. 10¾d.

EXERCISE 17.

1. $12116·12.
2. $98166·53.
3. $$752{\cdot}80\frac{1}{3}$.
4. $5732·49.
5. $1778·37.
6. $$972{\cdot}66\frac{1}{3}$.
7. $520·57.
8. $32218·08.
9. $409696·14.
10. $8903·07.
11. $$91{\cdot}78\frac{11}{47}$.
12. $$126{\cdot}13\frac{39}{149}$.
13. $24\frac{178}{7745}$.
14. $62\frac{1139}{11783}$.
15. $452·14.
16. $$33{\cdot}95\frac{9}{37}$.
17. $$51{\cdot}68\frac{9}{13}$.
18. $$86{\cdot}87\frac{1}{2}$.
19. $296·65.
20. $$123{\cdot}80\frac{11}{13}$.
21. $22728969.
22. $6340558.
23. $$9{\cdot}84\frac{71111099}{30314000}$.

EXERCISE 18.

1. 6016 cub. ft.
2. 103952 oz.
3. 93922 far.
4. 800453065 in.
5. 291454023 drs.
6. 2472 qts.
7. 13008552 lines.
8. 19060".
9. 5738 pts.
10. 503 in.
11. 10661357 sec.
12. 299024 mills.
13. 102635 grs.
14. 1742935 cub. in.
15. 745011 grs.
16. 34575 sq. ft. 108 in.
17. 5728½ in.
18. 2239 cub. ft.

19. 2856 qts.
20. 20721 scr.
21. 26304 oz.
22. 696984 in.
23. 595834¼ sq. yds.
24. 853437840 sec.
25. 3878 hrs.
26. 1899734 far.
27. 1112 pts.
28. 52248 grs.
29. 36136690704 sq. in.
30. 164695 far.

EXERCISE 19.

1. 1 mile 1 fur. 3 per. 3 yds. 7 in.
2. 4 days 6 hrs. 22 min.
3. 14 lbs. 1 oz. 5 drs. 7 grs.
4. 41 c. 2 c. ft. 10 cu. ft. 642 cu. in.
5. 31 ch. 5 bush. 2 pk. 1 pt.
6. 33 cub. yds. 23 cub. ft.
7. 1138 F. e. 3 qrs.
8. 12 wks. 5 days.
9. 3 tons 6 cwt. 3 qrs. 24 lbs. 1 oz. 4 drs.
10. 1 r. 3 per. 22 yds. 4 ft. 101 in.
11. 170 tuns 19 gal. 3 qts.
12. 5 yrs. 158 days 14 hrs. 34 min. 2 sec.
13. £178642 11s. 11¼d.
14. 298 E. $7 1 dime 4 cts. 9 mills.
15. 3 a. 1 r. 18 per. 15 yds. 2 ft. 16 in.
16. 136 c. 90 c. ft.
17. £3832 11s. 5d.
18. 4 lbs. 1 oz 19 dwt.
19. 221 bush. 1 pk. 1 gal.
20. 45833 F. e.
21. 8 tons 11 cwt. 1 qr. 24 lbs.
22. 3 miles 1 fur. 36 per. 5 yds. 1 ft.
23. 308° 38′ 31″.
24. 14 cub. yds. 8 ft. 781 in.
25. 1370 lbs. 3 drs. 2 scr. 7 grs.
26. 117 lbs. 10 oz. 5 dwt. 6 grs.
27. 11 cwt. 2 qrs. 16 lbs. 13 oz. 10 drs.
28. 9 a. 3 r. 8 per. 6 yds. 1 ft. 75 in.
29. 17 lea. 1 mile 2 fur. 7 per. 1 ft. 6 in.
30. 9930 fa. 2 ft. 2 in.

EXERCISE 20.

1. £1290 0s. 2¼d.
2. 195 miles 6 fur. 16 per. 2 yds. 1 ft. 6 in.
3. 281 lbs. 5 oz. 6 drs.
4. 212 cwt. 2 qrs. 12 lbs. 15 oz.
5. 160 yds. 2 ft. 1 in.
6. 179 bu. 1 pk. 1 gal.
7. 104 c. 4 c. ft.
8. 433 a. 2 r.
9. 28 yds. 2 qrs. 2 na. 2 in.
10. 154 lbs. 3 oz. 6 dwt. 18 grs.
11. 48 gal. 0 qt. 1 pt.
12. 226 sq. per. 7 yds. 5 ft. 36 in.
13. 100 wks. 4 days 3 hrs.
14. £266 14s. 9¼d.
15. 169 qrs. 2 lbs. 2 oz.
16. 178 Eng. ells 4 qrs.
17. 51 pks. 0 gal. 3 qts.
18. 440 oz. 10 dwt. 14 grs.
19. £478 5s. 1¼d.
20. 321 r. 28 per. 9 yds. 2 ft. 36 in.
21. 223 yrs. 27 wks. 3 days.
22. 287 lbs. 0 oz. 5 drs. 2 scr. 8 grs.
23. 281 a. 1 r. 11 per. 20 yds. 1 ft 103 in.
24. 73 lea. 2 miles 7 fur. 27 per. 2 yds. 2 ft. 10 in. 5 lines.

EXERCISE 21.

1. 98 days 4 hrs. 2 min.
2. 73 miles 0 fur. 16 per.
3. 1 qr. 15 lbs. 11 oz.
4. 481 gal. 2 qts. 1 pt.
5. 378 a. 1 r. 30 per.
6. £164 2s. 1½d.
7. 78 lbs. 9 oz. 7 dwt.
8. 78 yds. 3 qrs. 3 na.
9. 175 hrs. 54 min. 55 sec.
10. 177 c. 5 c. ft. 15 cub. ft.
11. 27 oz. 6 drs. 2 scr.
12. 4 fur. 39 per. 4 yds. 1 ft. 6 in.
13. 74 cwt. 0 qrs. 21 lbs.
14. 62 bush. 3 pks. 1 gal.
15. 69 years 47 weeks 4 days.
16. 59 r. 38 per. 24 yds. 2 ft. 30 in.
17. £57 6s. 6½d.
18. 80 oz. 18 dwt. 9 grs.
19. £84 19s. 6½d.
20. 52 cub. yds. 12 ft. 1461 in.
21. 82 hhds. 0 bar. 24 gal. 2 qt.
22. 73 drs. 1 scr. 8 grs.
23. 6 Flem. e. 0 qrs. 1 na.
24. 47 sq. per. 16 yds. 7 ft. 36 in.

EXERCISE 22.

1. 41 days 10 hrs. 16 min. 32 sec.
2. 73 qrs. 14 lbs. 12 oz.
3. 134 bush. 1 pk. 1 gal. 2 qts.
4. £2156 19s. 9d.
5. 125 gal. 0 qt. 1 pt.
6. 838 lbs. 1 oz. 10 dwt.
7. 298 miles 0 fur. 10 per.
8. 1148 years 81 days 23 hrs.
9. 508 cub. ft. 53 in.
10. 1343 c. 1 c. ft. 4 cub. ft.
11. 211 r. 9 per. 1 yd. 6 ft. 108 in.
12. 233 cwt. 3 qrs. 1 lb.
13. £1150 10s. 2½d.
14. 783 per. 1 yd. 0 ft. 1 in.
15. 57 sq. per. 23 yds. 6 ft.
16. 1975 yds. 3 qrs. 0 na. 1½ in.
17. 169 lbs. 4 oz. 7 drs. 1 scr.
18. 58 qrs. 8 lbs. 12 oz.
19. 557 miles 4 fur. 22 per.
20. 705 wks. 1 day 1 hr. 58 min.
21. 123 pks. 1 gal. 2 qts. 1 pt.
22. 1315 cwt. 1 qr. 7 lbs.
23. £1529 1s. 9½d.
24. 670 a. 3 r. 24 per. 5 yds. 6 ft. 108 in.

EXERCISE 23.

1. £1199 10s. 4d.
2. 1356 lbs. 0 oz. 6 dwt.
3. 339 days 17 hours 57 min.
4. 1321 Flem. ells 0 qrs. 3 na.
5. 3557 miles 1 fur. 32 per.
6. 5515 gal. 0 qt. 1 pt.
7. 8639 hours 26 min. 24 sec.
8. 1820 a. 1 r. 30 per.
9. 11116 oz. 4 drs. 2 scr. 19 grs.
10. £4897 1s. 4½d.
11. 16582 sq. per. 22 yds. 4 ft. 72 in.
12. 9208 cwt. 0 qr. 8 lbs.
13. 815 years 44 days 4 hours.
14. 1423 cub. yds. 9 ft. 1440 in.
15. 6806 bush. 2 pk. 1 qt.
16. £2780 0s. 3d.
17. 44042 per. 2 yds. 1 ft.
18. 102629 hours 5 min.
19. 3105 a. 2 r. 0 per. 18 yds. 4 ft. 72 in.
20. £516290 12s. 6d.

Exercise 24.

1. 3073 bush. 3 pks. 1 gal. 2 qts.
2. £14519 10s. $9\frac{3}{4}$d.
3. 2413 days 22 hours 31 min.
4. 3972 yards 1 ft. 5 in.
5. 1253 lbs. 4 oz. 19 dwt.
6. 6919 a. 2 r. 30 per. 25 yds. 7 ft. 72 in.
7. 19317 yds. 2 qrs. 1 na.
8. 155274 oz. 1 scr. 12 grs.
9. £79392 16s. $10\frac{1}{4}$d.
10. 4918 cwt. 1 qr. 22 lbs.
11. 4885 miles 7 fur. 5 per. 1 yd. 2 ft.
12. 3889 Eng. ells 2 na. 1 in.
13. 6307 cwt. 16 lbs. 5 oz. 10 drs.
14. 20146 a. 1 r. 6 per. 16 yds. 0 ft. 45 in.
15. 6575 years 146 days 11 hours 59 min. 6 sec.
16. 307 c. 3 c. ft. 2 cub. ft.
17. 23883 bush. 2 pks. 3 qts.
18. 65706 lbs. 2 drs. 2 scr. 5 grs.
19. £136344 16s. 3d.
20. 8610 lbs. 8 oz. 8 dwt. 13 grs.

Exercise 25.

1. £244 11s. 7d.
2. 11 cwt. 2 qrs. 10 lbs. 10 oz. $2\frac{10}{11}$ drs.
3. 24 days 16 hr. 4 min. $8\frac{4}{7}$ sec.
4. 76 bush. 3 pks. 1 qt. $0\frac{1}{3}$ pt.
5. 696 miles 5 fur. 8 per. 2 ft. 9 in.
6. 96 sq. per. 22 yds. 7 ft. 24 in.
7. 22 lbs. 3 oz. 7 drs. 1 scr.
8. 5 gals. 3 qts. $0\frac{1}{4}$ pt.
9. £1147 1s. $10\frac{5}{32}$d.
10. 79 cwt. 2 qrs. 14 lbs. 1 oz. $9\frac{9}{10}$ drs.
11. £8 3s. $7\frac{20}{24}$d.
12. 4 fur. 10 per. 4 yds. 1 foot $0\frac{30}{35}$ in.
13. 18 hrs. 16 min. $8\frac{28}{61}$ sec.
14. 17 lbs. 6 oz. 1 dwt. $9\frac{64}{108}$ grs.
15. 5 sq. per. 4 yards 3 feet $35\frac{59}{132}$ in.
16. 1 lb. 5 oz. $0\frac{45}{73}$ drs.
17. 8 yrs. 38 days 21 hrs. 2 min. 5 sec.
18. 10 oz. 1 dr. 1 scr. $8\frac{60}{97}$ grs.
19. 14 r. 14 per. 1 yard 5 feet $3\frac{105}{117}$ in.
20. 8 wks. 4 days 21 hrs. 18 min. $4\frac{673}{916}$ sec.
21. £826 18s. 5d. $\frac{105}{117}$ far.
22. 5 c. 3 c. ft. $2\frac{28}{85}$ cub. ft.
23. 48° 47′ $3\frac{56}{167}$″.
24. 5 French ells 2 qrs. 3 na. $2\frac{1}{1088}$ in.
25. 13 miles 5 fur. 18 per. 2 yds. 2 ft. $6\frac{42}{67}$ in.
26. £11 17s. $5\frac{3}{4}$d. $\frac{140}{161}$ far.
27. 9 lbs. 5 oz. 16 dwt. $11\frac{608}{653}$ grs.
28. 17 bush. 1 gal. $1\frac{90}{417}$ pt.
29. 4 days 12 hours 7 minutes $23\frac{300}{603}$ sec.
30. 11 cwt. 2 qrs. 2 lbs. 12 oz. $11\frac{135}{367}$ drs.

EXERCISE 26.

1. $763\frac{15594}{83682}$.
2. $77\frac{48529}{61296}$.
3. $120\frac{8168}{9475}$.
4. $153\frac{5679}{10281}$.
5. $207\frac{226107}{357820}$.
6. $103\frac{653}{5485}$.
7. $92\frac{1340}{1647}$.
8. $142\frac{1738}{4505}$.
9. $33\frac{3720}{7897}$.
10. $145\frac{23}{210}$.
11. $39\frac{387}{927}$.
12. $46\frac{168}{189}$.
13. $1917\frac{9180}{22900}$.
14. $154\frac{222684}{380163}$.
15. $24\frac{3499}{3919}$.
16. $18\frac{150}{289}$.
17. $23\frac{3}{59}$.
18. $4\frac{1675}{1855}$.
19. $1057\frac{929}{1607}$.
20. $87\frac{9039}{12162}$.

EXERCISE 27.

1. $4025\frac{1}{2}$ ft.
2. 176 lbs. 9 oz. 6 dwt. 6 grs.
3. DCCXIV ; MCXI ; MMDCCIV ; $\overline{\text{XC}}$MDCLXXI ; $\overline{\text{DCCCX}}$MMMCDLXXI ; $\overline{\text{XXX}}$MCMXVMMCLXIX.
4. £16 6s. $1\frac{1}{14}$d.
5. 160 lbs. 1 oz. 17 dwt. 12 grs.
6. One trillion two billions forty-seven thousand and six ; nine hundred quadrillions eleven trillions one hundred and ten billions eleven millions one hundred and ten thousand and eleven ; sixteen billions seven hundred and fourteen millions nine hundred and sixty-seven thousand nine hundred and four ; seventy-one billions three hundred millions four hundred thousand and two hundred.
7. 2 miles 2 fur. 26 per. 3 yds. 2 feet.
8. 7889 tons 2 cwt. 24 lbs ; $15778224.
9. 3 wks. 4 days 13 hrs. 10 min. $19\frac{45}{103}$ sec.
10. $792·05.
11. 12 lbs. 2 oz. 5 dwt.
12. $14·11.
13. 10001 ; 50500 ; 3333 ; 2650000 ; 89044 ; 450001709 ; 1094000 ; 1004900702.
14. 1626[illegible].
15. 6545238.
16. 7 a. 1 r. 31 per. 18 yards 6 ft. 36 in.
17. One £8 5s. and each of the others £4 3s.
18. 1st $124, 2d $155, and 3d and 4th each $232·50.
19. A $10·19$\frac{1}{11}$.
20. 5080 yds. 3 qrs. 1 na.
21. 3 tons 11 cwt. 3 qrs. 21 lbs 6 oz. 6[illegible] drs.
22. $6646\frac{1}{4}$ fathoms.
23. 35203.
24. 4600.
25. 9673.
26. $412·50 and $206·50.
27. 0606172.
28. 1101[illegible] bush.

29. .000007096405.
30. 772.
31. £7 19s. 1d.
32. 4675 gals.
33. 217 yards 2 feet.
34. A $374.80 and B and C each $187.40.
35. 1st receives 338 cub. ft., 2d 845 cub. ft., and 3d 1183 cub. ft.
36. 69 a. 13 per. 10 yds. 108 in.
37. 3 miles. 4 fur. 33 per. 5 yds. 6 in.
38. £78250 or $313000.
39. 413.
40. 114 lbs. 15 dwt. ; $837500.
41. $512\frac{607}{631}$ barrels.

Exercise 28.

1. 64.
2. 317.
3. 116.
4. 11.
5. 42.
6. They have none.
7. 29.
8. 117.
9. 75.
10. 1127.
11. 201.
12. They have none.
13. 23.
14. 13.
15. 3.
16. 17.
17. 58.
18. They have none.
19. 97.
20. 4.

Exercise 29.

1. 90.
2. 330.
3. 420.
4. 5040.
5. 720.
6. 3168.
7. 110880.
8. 4200.
9. 2520.
10. 1008.
11. 50400.
12. 12464.
13. 144.
14. 25200.
15. 1260.
16. 3465.
17. 1441440
18. 69300.
19. 166320.
20. 1920.

Exercise 30.

1. $\frac{77}{11}$, $\frac{99}{11}$, $\frac{297}{11}$, $\frac{440}{11}$.
2. $\frac{218}{109}$, $\frac{22563}{109}$, $\frac{47960}{109}$, $\frac{981}{109}$.
3. $\frac{2046}{93}$, $\frac{4371}{93}$, $\frac{6417}{93}$, $\frac{9300}{93}$.
4. $\frac{2821}{13}$, $\frac{7969}{13}$, $\frac{12051}{13}$, $\frac{62}{13}$.
5. $\frac{6696}{248}$, $\frac{75392}{248}$, $\frac{153016}{248}$, $\frac{23064}{248}$.
6. $\frac{127699}{611}$, $\frac{248677}{611}$, $\frac{482079}{611}$, $\frac{3055}{611}$.

EXERCISE 31.

1. $\frac{145}{7}$.
2. $\frac{101}{11}$.
3. $\frac{143}{10}$.
4. $\frac{358}{5}$.
5. $\frac{6461}{40}$.
6. $\frac{856}{31}$.
7. $\frac{5026}{27}$.
8. $\frac{40001}{200}$.
9. $\frac{440331}{617}$
10. $\frac{1558}{17}$.
11. $\frac{82}{13}$.
12. $\frac{23071}{11}$.
13. $\frac{573316}{929}$.
14. $\frac{7523}{18}$.
15. $\frac{6170181}{1000}$.
16. $\frac{123432}{1111}$.
17. $\frac{22909}{107}$.
18. $\frac{1279}{20}$.
19. $\frac{1517560}{647}$,
20. $\frac{366548}{191}$.

EXERCISE 32.

1. $17\frac{3}{4}$.
2. 154.
3. 1282.
4. $61075\frac{37}{43}$.
5. $37\frac{11}{19}$.
6. $28\frac{4}{7}$.
7. $17\frac{193}{643}$.
8. $130\frac{1291}{1632}$.
9. $25\frac{4}{87}$.
10. $157\frac{36}{93}$.
11. $1048\frac{107}{122}$.
12. $6\frac{11}{17}$.
13. $94\frac{35}{48}$.
14. $9\frac{6}{13}$.
15. $4\frac{35}{61}$.
16. $81\frac{1637}{6139}$.
17. $2670\frac{17}{23}$.
18. $123\frac{31}{92}$.
19. $109\frac{441}{832}$.
20. $2\frac{1483}{6431}$.

EXERCISE 33.

1. $\frac{7}{17}$.
2. $\frac{794}{1091}$.
3. $\frac{3307}{3331}$.
4. $\frac{31}{37}$.
5. $\frac{1961}{69}$.
6. $\frac{23}{3407}$.
7. $\frac{439}{1013}$.
8. $\frac{89}{97}$.
9. $\frac{47}{29}$.
10. $\frac{1697}{701}$.
11. $\frac{59}{113}$.
12. $\frac{101}{269}$

EXERCISE 34.

1. $\frac{10}{20}$, $\frac{15}{20}$, $\frac{16}{20}$, and $\frac{14}{20}$.
2. $\frac{252}{630}$, $\frac{270}{630}$, $\frac{280}{630}$, $\frac{420}{630}$, and $\frac{315}{630}$.
3. $\frac{49}{126}$, $\frac{108}{126}$, $\frac{70}{126}$, $\frac{104}{126}$, and $\frac{87}{126}$.
4. $\frac{180}{360}$, $\frac{330}{360}$, $\frac{315}{360}$, $\frac{210}{360}$, $\frac{308}{360}$, and $\frac{300}{360}$
5. $\frac{133}{180}$, $\frac{102}{180}$, $\frac{45}{180}$, $\frac{118}{180}$, $\frac{108}{180}$, and $\frac{111}{180}$.
6. $\frac{40}{120}$, $\frac{30}{120}$, $\frac{24}{120}$, $\frac{20}{120}$, $\frac{15}{120}$, and $\frac{12}{120}$.

7. $\frac{40}{60}$, $\frac{45}{60}$, and $\frac{50}{60}$.
8. $\frac{315}{360}$, $\frac{320}{360}$, $\frac{324}{360}$, $\frac{330}{360}$, $\frac{312}{360}$, and $\frac{340}{360}$.
9. $\frac{3553}{4199}$, $\frac{3458}{4199}$, and $\frac{3536}{4199}$.
10. $\frac{2880}{4080}$, $\frac{3672}{4080}$, $\frac{3825}{4080}$, and $\frac{2380}{4080}$.
11. $\frac{378}{420}$, $\frac{385}{420}$, $\frac{364}{420}$, and $\frac{320}{420}$.
12. $\frac{2772}{6930}$, $\frac{3960}{6930}$, $\frac{3780}{6930}$, $\frac{3080}{6930}$, $\frac{4620}{6930}$, $\frac{5445}{6930}$, and $\frac{5334}{6930}$.

EXERCISE 35.

1. $\frac{7}{68}$.
2. $\frac{4}{19}$.
3. $\frac{18}{61}$.
4. $\frac{5}{7}$.
5. 1.
6. $\frac{97}{99}$.
7. $19\frac{1}{2}$.
8. $1\frac{29}{80}$.
9. $\frac{9}{22}$.
10. $\frac{4}{33}$.
11. $1110\frac{123}{160}$.
12. $6\frac{2}{7}$.

EXERCISE 36.

1. $3\frac{1}{3}$.
2. $\frac{7}{99}$.
3. $\frac{44}{63}$.
4. $\frac{11}{28}$
5. $2\frac{3}{4}$.
6. $\frac{77}{680}$.
7. $\frac{306}{325}$.
8. $\frac{62}{133}$.
9. $\frac{13}{135}$.
10. $1\frac{112}{135}$.
11. $\frac{87}{143}$.
12. $\frac{512}{2775}$.

EXERCISE 37.

1. $\frac{1}{11}$ of a week.
2. $\frac{16}{29}$ of a qr.
3. $\frac{4}{15}$ of Flem. ell.
4. $126\frac{6}{19}$ per.
5. $\frac{49}{720}$ of a yd.
6. $\frac{3744}{19}$.
7. $\frac{1225}{20064}$.
8. $\frac{27}{770}$ of a £.
9. $\frac{95}{7392}$ of a week.
10. $\frac{33600}{323}$.
11. $\frac{22990}{1}$.
12. $\frac{3}{2800}$ of a £.

EXERCISE 38.

1. $\frac{137}{11100}$.
2. $\frac{47607}{223312}$.
3. $\frac{256259}{11626976}$.
4. $\frac{21}{64}$.
5. $\frac{175187}{825040}$.
6. $\frac{5}{464}$.
7. $\frac{17}{6272}$.
8. $\frac{84927}{104638}$.
9. $\frac{17}{335}$.
10. $\frac{27}{4840}$.
11. $\frac{457}{42660}$.
12. $\frac{7117}{12260}$.
13. $\frac{273}{6250}$.
14. $\frac{19789}{316880}$.
15. $\frac{476}{275}$.

EXERCISE 39.

1. 1 day.
2. 1 pk. 1 gal.
3. 1 bar. 10 gals. 2 qts. $\frac{239}{455}$ pt.
4. 9 oz.
5. 6 per. 7 yds. 0 ft. $92\frac{4}{7}$ in.
6. 2 na. $1\frac{37}{170}$ in.
7. 6s.
8. 35 a. 2 r. 20 per.
9. 13 miles 2 fur.
10. 2 cwt. 2 qrs. 16 lbs. 10 oz. $10\frac{2}{3}$ drs.
11. 2 oz. 3 drs. 2 scr. $16\frac{6}{7}$ grs.
12. 1s. $0\frac{39}{77}$d.

EXERCISE 40.

1. $2\frac{626}{693}$.
2. $1\frac{3083}{3465}$.
3. $3\frac{1}{20}$.
4. $13\frac{111}{140}$.
5. $243\frac{109}{420}$.
6. $38\frac{239}{630}$.
7. $233\frac{1005}{1463}$.
8. $1880\frac{9}{70}$.
9. $80\frac{107297}{135660}$.
10. $51\frac{86}{105}$.
11. $5\frac{3047}{4410}$.
12. $16\frac{21}{440}$.
13. $54\frac{127}{308}$.
14. $50\frac{54}{77}$.
15. $138\frac{8}{15}$.
16. $350\frac{29561}{47025}$
17. $4\frac{170144}{910910}$.
18. $81\frac{5}{84}$.
19. $5\frac{53}{590}$.
20. $56\frac{533}{616}$.

EXERCISE 41.

1. $\frac{75}{187}$.
2. $\frac{44}{45}$.
3. $\frac{183}{308}$.
4. $145\frac{43}{126}$.
5. $161\frac{5}{48}$.
6. $886\frac{271}{684}$.
7. $9\frac{167}{182}$.
8. $195\frac{1821}{4820}$.
9. $5\frac{2}{3}$.
10. $\frac{31561}{39270}$.
11. $40\frac{59}{1320}$.
12. $5\frac{52015}{87682}$
13. $8\frac{1903}{2940}$.
14. $85\frac{511}{1672}$.
15. $\frac{54908}{110979}$.
16. $158\frac{17}{504}$.

EXERCISE 42.

1. $\frac{6}{77}$.
2. $\frac{8}{77}$.
3. $2\frac{5}{11}$.
4. 8780.
5. $61\frac{1}{2}$.
6. $1\frac{29}{187}$.
7. $90\frac{5}{12}$.
8. $\frac{11}{39}$.
9. $2712\frac{18}{55}$.
10. $190\frac{13}{56}$.
11. $43\frac{133}{156}$.
12. $270\frac{174}{175}$.

13. $955\frac{1}{7}$.
14. 144.
15. $29\frac{3}{32}$.
16. $4\frac{135284}{840455}$.
17. $\frac{162}{847}$.
18. $\frac{16}{55}$.
19. $211\frac{1157}{1650}$.
20. $72\frac{409999}{853776}$.

EXERCISE 43.

1. $\frac{11}{14}$.
2. $\frac{24}{665}$.
3. $3\frac{1}{39}$.
4. $1\frac{23}{49}$.
5. $\frac{1071}{1216}$.
6. $40\frac{97}{189}$.
7. $2\frac{4}{19}$.
8. $5\frac{27}{28}$.
9. $1\frac{16813}{70111}$.
10. $52\frac{284}{405}$.
11. $\frac{22}{39}$.
12. $26\frac{5}{24}$.
13. $4\frac{569}{1144}$.
14. $\frac{128}{765}$.
15. $277\frac{349}{464}$.

EXERCISE 44.

1. £2 5s. $3\frac{8}{45}$d.
2. 17 bush. 2 pks. 3 qts. $0\frac{1}{8}$ pt.
3. 5 lbs. 14 oz. $4\frac{4}{7}$ drs.
4. 1 r. 31 per. 12 yds. 8 ft. $97\frac{5}{7}$ in.
5. 3 cwt. 24 lbs. 9 oz. $2\frac{3}{7}$ drs.
6. 2 a. 23 per. 25 yds. $83\frac{11}{33}$ in.
7. £3 13s. $10\frac{3}{4}$d. $\frac{11}{16}$ far.
8. 477 miles 7 fur. 30 per. 4 yds. $10\frac{11}{16}$ in.
9. 19 lbs. 7 oz. $1\frac{9}{11}$ drs.
10. 26 lbs. 9 oz. 0 drs. $6\frac{3}{5}$ grs.
11. 25 yds. 2 qrs. 1 na. $1\frac{3}{8}$ in.
12. 2 cwt. 10 lbs. 14 oz. $11\frac{7}{11}$ drs.
13. 33 bush. 3 pks. 2 qts. $1\frac{1}{3}$ pts.
14. 5 wks. 5 days 20 hours 9 min. $12\frac{18}{19}$ sec.
15. 1 lb. 9 oz. 18 dwt. 22 grains.
16. 3 a. 2 r. 5 per. 11 yds. 8 ft. 63 in.
17. £12 6s. $2\frac{8}{35}$d.
18. 2 cwt. 2 qrs. 6 lbs. 15 oz. $6\frac{506}{539}$ drs.

EXERCISE 45.

1. Twenty-seven hundredths; forty-three thousandths; seven thousandths; six thousand nine hundred and fourteen tenths of thousandths; eight thousand one hundred and ninety-six millionths; seventy-one thousand four hundred and twenty-three hundredths of millionths.
2. Six and seven tenths; ninety-three, and forty-two hundredths; one hundred and forty-seven, and one thousand three hundred and ninety-four tenths of thousandths; two hundred and seventeen, and nine tenths of millionths.

3. Seventy-one and eighty-nine hundredths of thousandths; one hundred and sixty-seven, and one hundred and ninety-three thousandths; ninety-one, and eight thousand six hundred and seventy-four tenths of millionths.

4. Five millions six hundred and seventy-four thousand three hundred and seventy-eight, and nine hundred and fourteen thousand seven hundred and eighty-six billionths; seventy-one millions three hundred thousand four hundred, and six hundred thousand four hundred and seven trillionths.

EXERCISE 46.

1. ·09; ·0076; ·000447.
2. ·00000700016.
3. ·0000005029011.
4. ·0087403.
5. ·709.
6. 496719·0011004.
7. 7691006·000014700930.
8. 1·71717.
9. 749·2000049.
10. 72·970704.

EXERCISE 47

1. 1742·4186.
2. 1274·05148.
3. 10185·028493.
4. 916·91238.
5. 1027·3333.
6. 11478·9156.
7. 161·0993.
8. 2011·84264.
9. 43·0445714.
10. 88·88038.
11. 161·09306.
12. 535·6531.

EXERCISE 48.

1. 705·753.
2. 92·14.
3. 44·73406.
4. ·07462746.
5. 9460·07567164.
6. 107·9055876.
7. 4·4082.
8. ·001648098.
9. 677·686149.
10. 10581·13056.
11. ·00727628.
12. ·0003444.

EXERCISE 49.

1. 72·922.
2. 11·684.
3. 4186·506.
4. 12742·178
5. 3·174.
6. 1·003.
7. 80·284.
8. 100077·510
9. 137·285.
10. 6·341.
11. ·893.
12. 2·324.

EXERCISE 50.

1. $\cdot\dot{2}8571\dot{4}$; $\cdot\dot{4}$.
2. $\cdot\dot{5}\dot{4}$; $1\cdot8$; $\cdot7$.
3. $\cdot\dot{9}2307\dot{6}$; $\cdot\dot{1}\dot{8}$; $\cdot95$.
4. $\cdot6958\dot{3}$; $\cdot\dot{5}7894736842105263\dot{1}$;
   $\cdot346938775510+$
5. $\cdot\dot{8}2716049\dot{3}$;
   $\cdot4419551934826883+$
6. $\cdot\dot{0}6\dot{3}$; $\cdot\dot{1}55080213903743\dot{3}$.
7. $\cdot544809228039+$;
   $31\cdot5862008965+$.
8. $\cdot\dot{8}6585\dot{3}$; $\cdot\dot{6}162\dot{3}$.
9. $\cdot8600372449908+$;
   $17\cdot0280154849+$
10. $12\cdot\dot{5}294117647058823$;
    $\cdot12499999886093+$

EXERCISE 51.

1. $\frac{1}{9}$; $\frac{26}{33}$; $\frac{4}{9}$.
2. $\frac{7}{33}$; $\frac{347}{999}$; $\frac{22}{101}$.
3. $\frac{2207497}{11111111}$; $\frac{3}{11}$.
4. $\frac{25}{198}$; $\frac{106}{495}$.
5. $\frac{2113}{9900}$; $\frac{13}{60}$; $\frac{2093}{9900}$.
6. $\frac{679}{5500}$; $\frac{1678}{9999}$.
7. $\frac{6043}{9000}$; $\frac{3176}{24975}$.
8. $\frac{1819}{1980}$; $\frac{47}{330}$.
9. $\frac{11113}{90000}$; $\frac{211}{1650}$.
10. $\frac{2783}{16650}$; 1 ; $\frac{86}{99}$.
11. $27\frac{43}{99}$; $17\frac{404}{495}$.
12. $467\frac{4111}{33300}$; $16\frac{3229}{19980}$.

EXERCISE 52.

1. $1\frac{59}{90}$.
2. $9\frac{419}{495}$.
3. $3\frac{1937}{4500}$.
4. $5\frac{20921}{36630}$.
5. $15\frac{971}{990}$.
6. $\frac{1876}{29403}$.
7. $\frac{11041}{17820}$.
8. $45\frac{1171}{22275}$.
9. $\frac{6140}{27639}$.
10. $\frac{54351}{116200}$.

EXERCISE 53.

1. $\cdot32738095$.
2. $\cdot6015625$.
3. $\cdot16449218$.
4. $\cdot31918777$.
5. $\cdot25351239$.
6. $\cdot56018$.
7. $\cdot84791\dot{6}\dot{3}$.
8. $\cdot12637$.
9. $\cdot0379585$.
10. $\cdot2137993$.
11. $\cdot8826025$.
12. $2\cdot7001379$.

EXERCISE 54.

1. 2s. 11d. ·9136 far.
2. 5 days 3 min. 28·224 sec.
3. 2 lbs. 1 oz. 6 drs. 6·72 grs.
4. 4 fur. 36 per. 3 yds. 6·048 in.
5. 3 r. 26 per. 17 yds. 5 ft. 124·31808 in.
6. 2 F. e. 3 na. 1·14318 in.
7. 9 hours 15 min. $52\frac{2}{3}$ sec.
8. 4 hhds. 1 bar. 13 gals. 2 qts. 1·7208 pts.
9. 3 r. 13 per. 13 yds. 5 ft. 73[illegible] in.
10. £0 18s. $3\frac{3}{4}$d. ·44 far.
11. £21 13s. $10\frac{1}{2}$d. ·328 far.
12. 9 years 53 days 13 hrs. 58 min. 19·92 sec.
13. \$6·343191.
14. 17 sq. yds. 6 ft. 129·492 in.
15. 2 tons 17 cwt. 1 qr. 2·38075 lbs.
16. £47 15s. $4\frac{1}{2}$d. ·049 far.

EXERCISE 55.

1. \$2377883·3333 +
2. 13860.
3. \$78·6425.
4. [illegible]
5. £1628 4s. $1\frac{1}{2}$d.
6. 17103[illegible].
7. A \$2099·45[illegible] and B and C each \$2249·62[illegible].
8. \$168720.
9. \$117099.
10. \$113·64[illegible].
11. DCCIV ; MCXI ; M$\overline{\text{X}}$DCCCLXXVI ; $\overline{\text{XX}}$MMMCDLXXI ; $\overline{\text{MXCXL}}$MMCCCLXXI.
12. 700000004020·0000006200019.
13. 22816.
14. $12\frac{6}{77}$.
15. ·0902777 +
16. 274.
17. [illegible]
18. 14791572 in.
19. 5 fur. 28 per. 4 yds 1 ft. ·024 in.
20. 4 yds. 2 ft. 2·566 in.
21. Greatest $\frac{4}{13}$ and least $\frac{6}{37}$.
22. $2\frac{1}{18}$ yds.
23. \$252·05$\frac{1}{3}$.
24. 002790 doz.; \$·119 and \$·107.
25. 110880.
26. 1170[illegible].
27. 65902.
28. 30 lbs. 6 oz. 15[illegible] drs.
29. 118·5904002.
30. [illegible]
31. \$7·688157.
32. $8\frac{1}{37}$.

EXERCISE 56.

1. 1[illegible] 6[illegible], 13, 24, 14[illegible], 5[illegible].
2. [illegible] 3[illegible], 4, 64[illegible], 2[illegible].
3. 3·5, ·642, 9, $0\cdot\dot{6}$, ·703, $5\cdot\dot{6}$.
4. 13, $5\cdot\dot{3}$, 5·6, 15·857, 20·181, 5·758.

5. Greatest 47 : 79, least 16 : 33.
6. Greatest 11 : 3, least 164 : 55.
7. Greatest 176 : 16·4, least 3 : ·89.
8. 11 : 16.
9. 27 ·187.
10. 12 : 1.
11. 12 : 5.
12. 2048 : 7245.

EXERCISE 57.

1. 120.
2. $3\frac{7}{13}$.
3. $4\frac{7}{11}$.
4. $151\frac{5}{8}$
5. 161.
6. $3\frac{1}{2}$.
7. $12\frac{2}{3}$.
8. 70.
9. $4\frac{13}{36}$.
10. $\frac{16}{23}$.
11. 9900.
12. $143·50.
13. £217 2s. $7\frac{1}{4}$d.
14. $445·80.
15. $30\frac{2}{5}$ days.
16. $24\frac{13}{31}$ weeks.
17. £10 17s. $2\frac{10}{17}$d.
18. $35·527.
19. $20·3903.
20. $136·30.

EXERCISE 58.

1. 40 a. 2 r.
2. $6·50.
3. 116 men.
4. $166\frac{2}{3}$ days.
5. $4·0514.
6. $8936·25.
7. $312·5518.
8. $125·11975.
9. $16787·40.
10. $1264·3717.
11. £37 0s. $3\frac{6}{13}$d.
12. $10\frac{1}{2}$ months.
13. £291 12s. $9\frac{11}{24}$d.
14. $219·77
15. $2·3625.
16. $311·48$\frac{4}{7}$.
17. $0·7379.
18. £1584 1s. $10\frac{313}{896}$d.
19. $165·0375.
20. $254\frac{5}{8}$.
21. $413·2421.
22. $4·4789.
23. $9\frac{3}{8}$ weeks.
24. 108 ft. $7\frac{1}{2}$ in.
25. 1005 miles 6 fur.
26. $583\frac{1}{3}$ yds
27. $53\frac{1}{3}$.
28. $50·46.
29. 184 a. 0 r. $26\frac{15}{19}$ p.
30. £5 13s. $7\frac{5}{31}$d.
31. 2a. 0r. $81\frac{8817}{9633}$ per.
32. $2·648.
33. $8·2776.
34. $71\frac{3}{11}$ lbs.
35. 9 days.
36. $104·5627
37. $811·652.
38. £7 17s. $6\frac{777}{3398}$d.
39. $1·8214.
40. $40\frac{17}{35}$.

EXERCISE 59.

1. $9\frac{13}{145}$ acres.
2. $712·72$\frac{8}{11}$.
3. $64\frac{8}{15}$ reams.
4. $2\frac{176}{245}$ days.
5. $20\frac{5}{98}$ days.
6. $38·65$\frac{160}{173}$.

7. $252\frac{1}{3}$ ft.
8. 7 men.
9. 36 days.
10. $169\frac{7}{17}$ cords.
11. 12 days.
12. 132 days.
13. 136 men.
14. $110\frac{151601}{160016}$.
15. $6277\frac{1}{2}$ bushels.
16. $7\frac{17441}{40041}$ acres.
17. \$235·$07\frac{69}{121}$.
18. $19237\frac{1}{2}$ lbs.
19. 10000 lbs.
20. $93\frac{13}{15}$ oz.
21. $1622\frac{184048}{207813}$ yds.
22. $182\frac{413}{3550}$ acres.
23. \$5456·25.
24. $195\frac{3}{4}$ bushels.

Exercise 60.

1. \$629·75.
2. \$2830·83.
3. \$696211·61.
4. £599 19s.
5. £2906 14s. $5\frac{3}{4}$d.
6. £90134 1s. $10\frac{3}{4}$d.
7. \$199175·$92\frac{1}{2}$.
8. \$26418·$40\frac{1}{13}$.
9. \$15371·$02\frac{11}{14}$.
10. £487 16s. $4\frac{1}{4}$d.
11. £56622 7s. $3\frac{4}{7}$d.
12. £14693 15s. $9\frac{89}{158}$d.
13. \$56435·$16\frac{3}{4}$.
14. \$21064·$58\frac{1}{13}$.
15. \$42·$90\frac{15}{64}$.
16. \$4289·$33\frac{11}{30}$.
17. \$1677·$75\frac{9}{16}$.
18. £46 15s. $10\frac{183}{244}$d.
19. \$219·$31\frac{13}{28}$.
20. £1540 8s. $6\frac{8}{55}$d.
21. £24338 4s. $9\frac{5}{8}$d.
22. £10771 4s. 6d.
23. \$30507·40.
24. £191 14s. 7d.
25. \$9353·$03\frac{485}{568}$.
26. \$70·$91\frac{23}{37}$.
27. \$755·$41\frac{6}{13}$.
28. £274 10s. $7\frac{21}{53}$d.
29. \$20954·$12\frac{18}{115}$.
30. £1749 6s. $3\frac{187}{284}$d.

Exercise 61.

1. ·09 ; ·045.
2. ·037 ; ·2025.
3. ·062 ; ·082.
4. 1·11 ; 1·47.
5. ·0975 ; ·6316.
6. ·08 ; ·005.
7. ·00375 ; ·02625.
8. ·002 ; ·09375.
9. ·162 ; ·0098.
10. 1·472 ; ·2612.
11. 7 ; 61.
12. 147 ;
13. $87\frac{1}{2}$; 220.
14. 111 ; 110·7.
15. 106 ; ·7.
16. $67\frac{1}{2}$; $8\frac{1}{4}$.
17. $1\frac{3}{8}$; $1\frac{7}{8}$.
18. ·05 ; 121·7.
19. $\frac{1}{4}$; $13\frac{1}{2}$.
20. $\frac{1}{16}$; $27\frac{1}{8}$.

Exercise 62.

1. \$1644·516.
2. \$1079·75.
3. \$5·5875.
4. \$524·72.
5. 17 a. 1 r. 22 per. 17 yds. 7 ft. $25\frac{13}{19}$ in.
6. 1 gal. 2 qts. $1\frac{1}{10}$ pts.
7. 3 lbs. 1 oz. 2 dwt $22\frac{58}{61}$ grs.

8. 14 y. [illegible] 1 day 27 min.
9. £5 12s. $1\frac{72}{100}$d.
10. $854·74[illegible].
11. $65·6807.
12. $357·6995.
13. $1114·35; $1262·93; $2154·41; $668·61; $2228·70.
14. 45·4a. wheat; 40·86 a. grass; 38·59 a. peas; 43·13 a. oats; 18·16 a. root crops; 40·86 a. fallow.
15. $263\frac{81}{100}$ killed or wounded; $80\frac{29}{100}$ taken prisoners.

EXERCISE 63.

1. $3·551.
2. $71·052.
3. $162.
4. $288·52595.
5. $33·94½.
6. $587·95⅝.
7. $11·95.
8. $344·4444.
9. $65·6768.
10. $104·165625.
11. $131·11¼.
12. $374·10.
13. $328·70½.
14. $546.
15. $136·17.
16. $4807·68¾.
17. $186·25.
18. $38·47½.

EXERCISE 64.

1. $21·71015.
2. $179·68½.
3. $284·80.
4. $32·77½.
5. $7·55⅝.
6. $140·59¼.
7. $65·72½.
8. $714·74.
9. $186·78¾.
10. $153·094375.

EXERCISE 65.

1. $676·3326.
2. $8575·78.
3. $1219·21875.
4. $1956·84552.
5. $5693·60.
6. $1882·3529.
7. $5511·81102.
8. $5648·80.
9. $683·4375.
10. $9797·3563.

EXERCISE 66.

1. $107·14.
2. $1057·707.
3. $182·656.
4. $124·432.
5. $10·1235.
6. $77·4427½.
7. $413·86⅔.
8. $4617·291⅔.
9. $4162·744.
10. $292·4128.
11. $6669·4375.
12. $5533·50.
13. $120·7732[illegible].
14. $51·88895⅔
15. $11·4467155[illegible]
16. $159·721481[illegible]

Exercise 67.

1. $·04 ; $·035 ; $·055.
2. $0·165.
3. $0·98.
4. $0·355.
5. $0·665.
6. $0·65.
7. $0·265.
8. $0·377.
9. $0·1955.
10. $0·2766̇.
11. $0·10916̇.
12. $0·15783̇.

Exercise 68.

1. $885·36.
2. $81·066.
3. $88·6908.
4. $41·69428¼.
5. $12·96295¼.
6. $201·9515.
7. $10·7916.
8. $21·585804¾.
9. $115·85069¼.
10. $202·73545¼.
11. $514·31488¾.
12. $579·91816¾.
13. $38·1616.
14. $6·35026¾.
15. $12·03615.
16. $1356·7605.

Exercise 69.

1. $225·043.
2. $209·9815.
3. $304·403.
4. $101·9151.
5. $36·8807.
6. $111·0229.
7. Am't = $621·1484.
   Inter. = $121·1484.
8. Am't = $477·0074.
   Inter. = $77·0074.
9. Am't = $871·0362.
   Inter. = $156·1362.
10. Am't = $906·7706.
   Inter. = $11[illegible] 1706.

Exercise 70.

1. $12·7273.
2. $1·3301.
3. $7·2816.
4. $65·4188.
5. $29·40.
6. $15·3095.
7. $70·1920.
8. $75·8843.
9. $00·6587
10. $8·1879.
11. $9·5416.
12. $77·4363.

Exercise 71.

1. $6·125.
2. $11·20.
3. $13·49⅐.
4. $7·525175.
5. $11·5659.
6. $0·005¾.

EXERCISE 72.

1. A's gain =$\$655\cdot71\frac{3}{7}$.
B's " = 1020.
C's " = $24\cdot28\frac{4}{7}$.
2. B's loss =$542·82.
C's " = 361·88.
3. First =$\$75\cdot84\frac{10}{17}$.
Second = 61·40.
Third = $169\cdot75\frac{5}{17}$.
4. A's share=$1498·60.
B's " = 2622·55.
C's " = 3371·85.
5. $\$1381\cdot57\frac{17}{19}$; $\$1973\cdot68\frac{8}{19}$; $\$4144\cdot73\frac{13}{19}$.
6. A's share=$\$1689\cdot88\frac{63}{89}$.
B's " = $1510\cdot11\frac{26}{89}$.
7. A's " =$\$1837\cdot98\frac{788}{1469}$.
B's " = $2573\cdot17\frac{1327}{1469}$.
C's " = $4588\cdot83\frac{873}{1469}$.
8. A's loss =$5325.
B's " = 2130.
C's " = 3195.
9. $\$1011\cdot52\frac{8}{171}$; $\$1618\cdot43\frac{47}{171}$; and $\$2312\cdot04\frac{116}{171}$.
10. A should have $\$5358\cdot17\frac{601}{1047}$, and B $\$2131\cdot82\frac{446}{1047}$.

EXERCISE 73.

1. A's share=$\$2031\cdot02\frac{262}{419}$
B's " = $2568\cdot97\frac{157}{419}$.
2. A's share=$\$633\cdot96\frac{12}{53}$.
B's " = $896\cdot60\frac{20}{53}$.
C's " = $869\cdot43\frac{21}{53}$.
3. A's share=$\$772\cdot72\frac{8}{11}$.
B's " = $1227\cdot27\frac{3}{11}$.
4. A's share=$\$44\cdot38\frac{26}{73}$.
B's " = $41\cdot09\frac{43}{73}$.
C's " = $34\cdot52\frac{4}{73}$.
5. A's share=$\$753\cdot42\frac{34}{73}$.
B's share=$\$342\cdot46\frac{42}{73}$.
C's " = $513\cdot69\frac{63}{73}$.
D's " = $890\cdot41\frac{7}{73}$.
6. A's share=$\$198\cdot50\frac{50}{67}$.
B's " = $389\cdot92\frac{36}{67}$.
C's " = $361\cdot56\frac{48}{67}$.
7. A's share=$\$2222\cdot22\frac{2}{9}$.
B's " = $777\cdot77\frac{7}{9}$.
8. A's share=$\$7205\cdot71\frac{3}{7}$.
B's " = $2177\cdot55\frac{5}{49}$.
C's " = $316\cdot73\frac{23}{49}$.

EXERCISE 74.

1. $\$15\cdot67\frac{1}{2}$.
2. $11·57.
3. $27·30.
4. $628·10.
5. $88·05.
6. $\$28\cdot57\frac{1}{2}$.
7. $282·87.
8. $\$37\cdot36\frac{1}{2}$.

EXERCISE 75.

1. $322·30.
2. $1041·30.
3. $712·908.
4. $555·94.
5. $2413·5975.
6. $1218·56.
7. $5640·264.
8. $4668·6636.

EXERCISE 76.

1. $9\frac{3}{8}$ per cent.
2. 12[illegible] per cent.
3. 23[illegible] per cent.
4. 5[illegible] per cent.
5. 9[illegible] per cent.
6. 15[illegible] per cent.

EXERCISE 77.

1. $1·4406.
2. $251·1627.
3. $5·6179.
4. $10\frac{13}{43}$ cts. per lb.
5. $\$6636{\cdot}\dot{3}\dot{6}$.
6. $7·39805.
7. $133·0275.
8. $9·1603 per 1000.

EXERCISE 78

1. 66 cents.
2. 364 lbs.
3. $57\frac{9}{13}$ yards.
4. $\$3{\cdot}61\frac{1}{14}$.
5. $\$6{\cdot}75\frac{9}{32}$.
6. $0·072[illegible].
7. $3·00.
8. 202·304 quarts.

EXERCISE 79.

1. £178 14s. $7\frac{3}{4}$d.
2. £365 19s. $2\frac{3}{4}$d.
3. £183 7s. $2\frac{3}{4}$d.
4. £17 11s. $4\frac{5}{32}$d.
5. $\$1175{\cdot}46\frac{3}{4}$.
6. $\$785{\cdot}59\frac{1}{4}$.
7. $1059·09375.
8. $\$358{\cdot}37\frac{1}{2}$.
9. £204 7s. $6\frac{1}{4}$d.
10. £184 5s. 9d.
11. £209 11s. $11\frac{1}{4}$d.
12. £18 8s. 9d.
13. $144·77804.
14. $1434·84229.
15. $2004·37255.
16. $4105·07115.
17. $\$785{\cdot}577\frac{1}{2}$.
18. £605 13s. 5d.
19. £516 8s. [illegible]d.
20. $2292·2809.

EXERCISE 80.

1. 7 times.
2. $5\frac{1}{2}$ "
3. $16\frac{1}{2}$ "
4. $3\frac{1}{2}$ "
5. 4 times.
6. $20\frac{1}{2}$ "
7. $13\frac{1}{2}$ "
8. $6\frac{3}{4}$ "
9. 13 times.
10. $5\frac{1}{2}$.
11. $79\frac{1}{2}$.
12. 210.

13. 75.
14. $50\frac{1}{2}$ lbs.
15. $69.
16. 33.
17. $7\frac{1}{3}$ days.
18. $3\frac{9}{137}$ days.
19. 70 days.
20. $105.
21. $7500.
22. $1·31$\frac{6}{19}$.
23. $3·92$\frac{29}{37}$.
24. 84.
25. $25\frac{5}{7}$ feet.
26. In $15\frac{5}{13}$ hours and $46\frac{2}{13}$ m., or $53\frac{11}{13}$ miles from where they started.
27. 200 miles.
28. $109\frac{9}{19}$.
29. 88.
30. 640.

EXERCISE 81.

1. 289.
2. 12167.
3. 77841.
4. 531441.
5. 1296.
6. 3125.
7. 4096.
8. 2187.
9. 256.
10. 19683.
11. 343.
12. 14641.
13. 531441.
14. $\frac{32}{3125}$.
15. 1500625.
16. 23396569.
17. $76\frac{49}{64}$.
18. 24389.
19. $50\frac{46}{81}$.
20. 59049.

EXERCISE 82.

1. 86.
2. 43.
3. 126.
4. 231.
5. 378.
6. 999.
7. ·094.
8. ·629.
9. 27·5.
10. 106·759.
11. 313·240.
12. 2590·929.
13. 958·523.
14. 20·7304.
15. $\frac{3}{4}$, $\frac{8}{9}$, $\frac{11}{13}$, $\frac{9}{50}$.
16. ·7977, ·7276.
17. 20·698.
18. 25·095.
19. 1062·024.
20. 462·383.

EXERCISE 83.

1. 32.
2. 87.
3. 24.
4. 63.
5. 99.
6. 125.
7. 251.
8. 364.
9. 626.
10. 8·82.
11. 99·7.
12. ·971.
13. $\frac{2}{5}$, $\frac{5}{13}$, $\frac{7}{9}$, $\frac{4}{5}$.
14. ·971, ·965.
15. 1, ·464, ·4807.
16. 75·36.
17. 9·346.
18. 97·158.
19. 200·92.
20. 3·0275.

EXERCISE 84.

1. $275·67$\frac{27}{30}$.
2. 122.
3. $1177·27¼.
4. 29·91.
5. $3\frac{3}{770}$.
6. $1004·0968.
7. $\frac{7}{9}, \frac{19}{45}, \frac{389}{1650}, \frac{293}{333}$.
8. $5·12.
9. $\frac{5}{9}$ times 8.
10. 44·169.
11. $714883.
12. 17223 lbs. 11 oz. 2 dwt. 21⅓ grs.
13. $\frac{1}{3}, \frac{1}{7}, \frac{3}{8}, \frac{19981}{33732}$.
14. 5 fur. 28 per. 3 yds. 2 ft. $2\frac{10}{33}$ in.
15. 33597·4749.
16. $3471·38$\frac{82}{121}$.
17. $10507·11$\frac{33}{650}$.
18. $39·141$\frac{3}{5}$; $37·19807.
19. $30\frac{19}{37}$ days.
20. 12·2177.
21. 1 wk. 5 days 19 hours 41 min. $32\frac{4}{13}$ sec.
22. $7910·93¼.
23. 109·708.
24. XXVII; CCCXCIII; M$\overline{\text{V}}$DCC; $\overline{\text{LXXV}}$MMMCMIV; $\overline{\overline{\text{MX}}\text{CXXXV}}$MDCCXVIII.
25. 2178 : 85.
26. They have none.
27. $46\frac{13}{25}$ hours.
28. 1192¼ rods.
29. $3843·64226; £613 5s. 0¼d.
30. A't =$954·01488. Int.=$154·01488.
31. $3\frac{1}{3}$, $1\frac{4}{9}$, and $\frac{11}{18}$ per cent.
32. 79200.
33. 901040000007008·00000009030017.
34. A's share, =$844·17$\frac{5}{7}$. B's do.=$719·11$\frac{3}{7}$. C's do.=$906·70$\frac{7}{8}$.
35. $4\frac{417}{567}$ per cent.
36. 1 a. 1 r. 33 per. 9 yds. 3 ft. 80 in.
37. £890 6s. 9¾d.; $78·52⅞.
38. 2 bu. 2 pks. 1 qt. 1 pt.
39. 19·26.
40. $31\frac{11}{19}$ minutes.
41. 78·075.
42. 8928 a. 3 r. 15 per. 2 yds. 2 ft. 36 in.
43. 10039¼ inches.
44. $6\frac{173873}{1120000}$.
45. $1144·21⅓.
46. $2604.
47. $109241\frac{11}{13}$ paces.
48. $6801·72$\frac{13}{18}$.
49. $392·220⅝.
50. 9⅔ days.
51. $\frac{2304}{16785}$.
52. $57\frac{3197}{4175}$.
53. 18480.
54. $251·714¼.
55. $49\frac{13}{18}$ cents.
56. $53·125.
57. A's share, =$70·75$\frac{1234}{1333}$. B's " =$46·57$\frac{934}{1331}$. C's " =$82·67$\frac{1121}{1333}$.
58. $61\frac{31}{171}$ cents.
59. A and B have each $137½, C has $275, and D $550.
60. $9\frac{7}{27}$ hours per day.

THE END.

LT001.511.522 E4 1866

Mis Margret ~~[illegible]~~

Mr

Prepair for Mrs J. Ann

℞

1 dram Oil Annis ʒi.

3 oz Antim Vinum ʒiii.

4 oz Ipecacuana Vinum ʒii

4 oz Tinct Benzoin ʒii

6 oz Syrup Squills ʒvi

4 oz Spirits Nitric Eath

4 oz — " — Sulphuric — " —

1 oz Gum Arabic Powd.

Mix dose ½ Teaspoon

two a Teaspoonful.

4 or 5 times a day

[illegible]

[illegible] M. Marge

4 oz Tinct Peruvian Bk
4 oz — " — Gentian
4 oz — " — Rhei
℞ 2 oz — " — Columbo
2 oz — " — Quassia
4 oz Ext Dandelion
4 oz — " — Ginger
Mix

Take one Teaspoon 3 times a day in a Little Water

A. J. Case

[illegible]

LOVELL'S SERIES OF SCHOOL BOOKS.

HAVING [illegible] EDUCATIONAL Works [illegible] for our COLONIAL SCHOOLS, [illegible] the supply of [illegible] with success, his Books [illegible] in the Schools, and [illegible] the following [illegible] SCHOOL BOOKS, already issued.

LOVELL'S GENERAL GEOGRAPHY, [illegible] Beautiful Engravings, [illegible] World. By J. GEORGE HODGINS, LL.D. [illegible]

[illegible] This Book is [illegible] Academy, [illegible] should [illegible]

EASY LESSONS IN GENERAL GEOGRAPHY, with Maps and [illegible]; being [illegible] General Geography. By the [illegible]

A SCHOOL HISTORY OF CANADA, and [illegible] N. A. PROVINCES [illegible]

[illegible]